Original Research Article

Analyzing the Use of Fresnel Lenses to Optimize Solar Sails

Vrishank Ram[1*], Abhinav Shivalli[2*], Edwin Mu[3*]

[1]Archbishop Mitty High School, 5000 Mitty Way, San Jose, CA 95129, United States;
[2]American High School, 36300 Fremont Blvd, Fremont, CA 94536, United States;
[3]Los Altos High School, 201 Almond Ave, Los Altos, CA 94022, United States

ABSTRACT

Fresnel lenses are segmented convex lenses with a reduced mass that can collimate light and direct it at a focal point, and have proven useful in solar concentrator technologies like solar cells. In this study, we analyze the feasibility of using Fresnel lenses in solar sails (devices that utilize light from the Sun as a form of stellar propulsion). A major challenge for solar sails is maximizing surface area while minimizing mass, which increases propulsion. By focusing and capturing light over a larger surface while being less massive, Fresnel lenses could improve the area-mass ratio (AMR), and this investigation analyzes whether they increase the efficiency of solar sails. After designing the lens and sail, we used ANSYS Lumerical FDTD and other simulation methodologies to model light interaction with the lens, measuring lens material efficiency through the intensity of the focused light. The results provided a theoretical design with a theoretical AMR nearly double that of NASA's ACS-3 solar sail; once optical losses are applied to both designs on the same basis, the improvement is more modest. These idealized values hold only when the physical limits of manufacturing extremely thin lenses are not considered. Current works focus on novel surface materials, increased thermal efficiency, or structural design changes for efficiency. Through this work, we present an analysis of AMR improvement through the integration of Fresnel lenses into solar sails, with quantitative comparisons to designs such as the ACS-3, contributing to the limited existing literature on this application.

Keywords: AMR; NASA's ACS-3; Photonic Propulsion; Solar Sail; Fresnel Lens

INTRODUCTION

Fuel has always been a major barrier to space travel, especially long-distance space travel. Solar sails are an attractive solution, utilizing light from the Sun to propel themselves forward by reflecting photons using a mirror-like surface (1). Because the Sun constantly radiates light, it is an endless propulsion energy source for a solar sail with distance that does not scale with weight. Several satellites and missions have used solar sails, including the ACS-3 solar sail launched in April 2024 (1).

A solar sail works by reflecting photons off its surface, transferring the forward momentum of the photons to the solar sail due to the conservation of momentum. Therefore, the more photons reflected, the more propulsion the solar sail receives. Solar sail propulsion efficiency is limited by the ratio between the sail's reflective area and mass, referred to as the area-mass ratio (AMR) (2). This is a challenge for solar sails because solar sails currently accelerate slowly and require large surface areas (2), so more efficient solar sails would improve applications in interstellar space

Corresponding author: Vrishank Ram, E-mail: vriram09@gmail.com.

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travel (1), which is the goal of this study.

A Fresnel lens achieves a lower mass compared to traditional lenses by removing excess material that is not at the surface of the lens. This allows it to keep the curvature of the original lens (in turn retaining the focal properties) while compromising image quality due to sharp edges (3). Fresnel lenses are still being applied in modern technology, such as in active research into more efficient solar cells (4) and light concentrators (5). Due to its lightweight properties, this study investigates whether Fresnel lenses could be used in solar sails to increase efficiency.

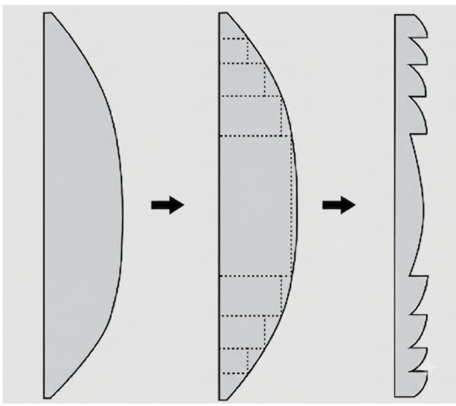

***Figure 1.* Geometric comparison of Fresnel and standard lenses (3).** *A Fresnel lens maintains equal refractive performance to a traditional lens by preserving surface curvature in sections while greatly reducing material thickness and mass. Image used under CC BY-NC 3.0.*

Using Fresnel lenses with solar sails is a mostly unexplored field of study (6). Of the small amounts of past research, one design uses one large Fresnel lens that focuses light on small mirrors (6).

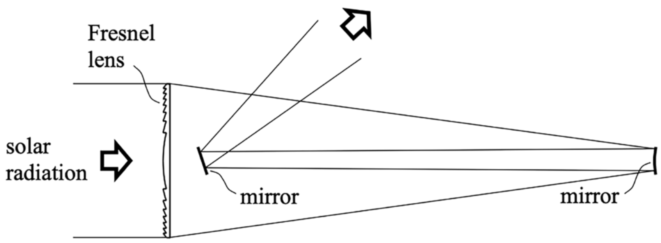

***Figure 2.* Past Fresnel Lens Design Proposal (6).** *The design utilizes a Fresnel lens to focus light onto smaller mirrors than current solar sails.*

The potential materials that can be used for this application of Fresnel lenses include CP-2 (Kapton) and Mylar. These materials were chosen for their refractive index and low density, making them great for high AMR requirements. They are also durable and commonly used in aeronautical applications, making this design feasible for usage in space if constructed as well as easily accessible material-wise. Kapton has a density of 1420 kg/m^3 and a refractive index of 1.751 (7) and has been explored in previous studies on Fresnel lenses for space applications (5). On the other hand, Mylar is currently used in solar sails (8) and has a density of 1390 kg/m^3 and a refractive index of 1.66 (7).

The ACS-3 solar sail is the most recent solar sail launched by NASA in April of 2024, making it ideal for a theoretical comparison (1) (Figure 3). This solar sail had a thickness of 2.115 microns, four 7-meter-long booms, a total area of 80 m^2, and a sail mass of 0.34 kg (9). Using these numbers, the total boom length, the length of the supporting beam that holds the sail up, can be calculated to be 28 meters and the area-mass ratio (AMR) would be ~235 m^2/kg. However, because the value of the sail mass differs from the result given by this paper's method of calculation (volume multiplied by density, likely failing due to neglecting the mass of the metal coating), this paper used a theoretical ratio predicted by our methods, giving a theoretical AMR of ~340 m^2/kg.

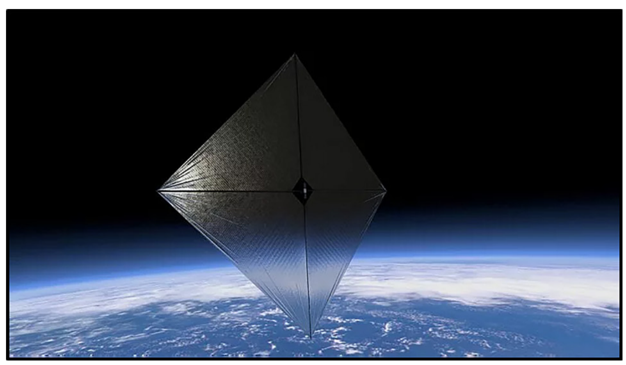

***Figure 3.* Orbital operation of the ACS3 solar sail (1).** *Solar sails utilize the momentum of solar photons to generate constant thrust, enabling propellant-free propulsion. Image in the public domain; courtesy of NASA.*

METHODS AND MATERIALS

Experimental Design

A conceptual design was used to test the efficiency of using Fresnel lenses in solar sails with similar structure to the ACS-3 for consistency in design and accuracy in comparison. The design was aimed at maximizing momentum transfer from photons to the solar sail. Momentum transfer is optimized when the largest component of the photons' momentum is along the axis of the solar sail's path of travel (i.e. when the dot product of the photons' momentum and the velocity of the solar sail is the greatest). In the previous design mentioned in Figure 2, maximum momentum transfer would be achieved with the photons hitting the mirror perpendicular to the surface, which is not the case. But when light reflects off a surface, the reflected angle propagates at the same angle perpendicular to the surface (i.e. the angle of incidence is the same as the angle of reflection). Exploiting this property, we can derive that a flipped Fresnel lens that intersects the path of the reflected ray is all that is necessary. This result implies that parallel light rays can be reflected in the direct opposite direction with Fresnel lenses, thus optimizing the momentum transfer (Figure 4A). In addition to this design, booms were included to support the Fresnel lens attachment and to keep a consistent contribution of mass by external components for accurate comparison later in the study. Further detail is discussed in the Optimization Methodology section.

In the design, although most sails currently being made are square, there is still interest and research into circular sails (10). The sail was decided to be circular because Fresnel lenses are made up of concentric circles, thus circular designs make manufacturing easier. A circular design also ensures consistency in capturing light from all angles, thus keeping acceleration of the overall sail consistent (Figure 4C).

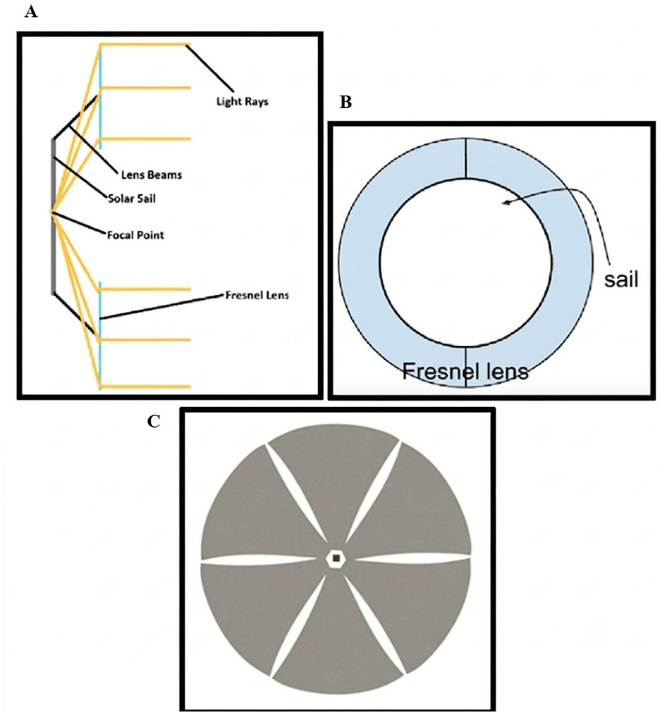

Figure 4. ***Proposed design of the Fresnel lens solar sail system.*** *(A) Side-view ray diagram: incoming sunlight is refracted by the Fresnel lens, converges at the focal point, and exits anti-parallel to the incident beam, maximizing the photon momentum transferred along the sail's thrust axis. (B) Front-view schematic showing the concentric-ring Fresnel lens suspended coaxially above the circular reflective sail membrane, connected via six radial booms. (C) Circular sail layout: the sail uses six radial booms instead of the four diagonal booms of a square sail, maximizing surface area and keeping the membrane tensioned. Image (C) courtesy of NASA/JPL.*

Optimization Methodology

After finalizing the design, the investigation looked at all the relevant variables to optimize the AMR. For this experiment, the AMR can be defined as the total surface area of the sail in m^2 divided by the total mass of the apparatus in kg. The main variable which we used to optimize the ratio was the radius of the solar sail, r (units meter). All the variables used in this investigation are listed below.

R = radius of curvature of Fresnel lens (m)
r = radius of solar sail (m)
w = diameter of Fresnel lens (m)
f = focal length of the lens (m)
n = refractive index of Fresnel lens material. Kapton: 1.751, Mylar: 1.66
d_w = density of Fresnel lens material (kg/m^3) (Kapton: 1420 kg/m^3 or Mylar: 1390 kg/m^3),
d_r = density of solar sail material (kg/m^3) (Mylar: 1390 kg/m^3)
h = thickness of solar sail and maximum thickness for Fresnel lens (m) (2.115×10^{-6} *m*)

To ensure a practical lens design, parameters were placed on this investigation. Firstly, for the solar sail to be comparable to ACS-3, the solar sail should use the same amount of boom length (as mentioned in the methods) to limit the size of the sail, which is 28m.

Boom Length and Geometry

To find the boom length in terms of the above variables, since the solar sail is circular, there will be

6 boom lengths of r and 6 boom lengths from the edge of the sail to the Fresnel lens. The choice of a 6-boom solar sail ensures structural rigidity and uniform tensioning of the solar sail due to the formation of a 6 section equilateral triangle structure, which remains due to the structural nature of triangles. These booms allow a stable platform to attach the lens to the sail. Because it would be more stable if the boom is connected to the center of the Fresnel lens and to the same spots where the boom lengths of the sail were attached, the distance away from the solar sail in the front view would be $w/2$ but the Fresnel lens is also f away from the sail in the side view as that is the distance from the focus and the lenses. Using the Pythagorean Theorem, the boom length connecting the Fresnel lens to the solar sail equals $\sqrt{\frac{w^2}{4}+f^2}$. Now, as there are 6 of them, as well as the 6 boom lengths of r for the solar sail itself, the total boom length would be $6r + 6\sqrt{\frac{w^2}{4}+f^2} = 28$ (Eqn. 1).

Before solving this equation, we can also define some more parameters. Visually, the radius of curvature R is just the radius of the Fresnel lens, which is the addition of the radius of the solar sail r with the length of the Fresnel lens w. As such, $R = w + r$.

Relating Radius of Curvature (R) with Radius of Solar Sail (r)

To find the relationship between the focal length f and R, the Lens Maker's Equation can be used:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

This equation depends on the refractive index, therefore the equations must be separately calculated for both materials as each has a different n. However, because a Fresnel lens has an R_2 of infinity due to its plano-convex shape, it allows us to use the thin lens approximation, where $\frac{1}{f} = (n-1)\left(\frac{1}{R}\right)$. Knowing this, f can be defined in terms of R: $f = \frac{R}{(n-1)}$. The boom constraint equation, $6r + 6\sqrt{\frac{w^2}{4}+f^2} = 28$, can be rewritten as $\frac{w^2}{4} + f^2 = \left(\frac{28-6r}{6}\right)^2$ through algebraic manipulation. Since f is defined as $\frac{R}{(n-1)}$, the equation becomes $\frac{w^2}{4} + \frac{R^2}{(n-1)^2} = \left(\frac{28-6r}{6}\right)^2$. Given $R = w + r$, we can eliminate the variable w by substituting $w = R - r$: $\frac{(R-r)^2}{4} + \frac{R^2}{(n-1)^2} = \left(\frac{28-6r}{6}\right)^2$. Expanding the expression and factoring, we get $R^2\left[\frac{1}{(n-1)^2} + \frac{1}{4}\right] - \frac{Rr}{2} + \frac{r^2}{4} - \left(\frac{28-6r}{6}\right)^2 = 0$. Taking a closer look reveals that the expression is a quadratic in terms of R. If we define the terms $a = \frac{1}{(n-1)^2} + \frac{1}{4}, b = -\frac{r}{2}, and\ c = \frac{r^2}{4} - \left(\frac{28-6r}{6}\right)^2$, R can be solved for using the quadratic equation. Thus, $R(r) = \frac{-b+\sqrt{b^2-4ac}}{2a}$ (Eqn. 2).

One limit we do have to place is the limit of the radius of the solar sail (r). This is due to the fact that the sun is of a considerable size such that the focused image due to the light from the sun cannot be considered a point, but rather a small image (Figure 5). If the sail itself is larger than this image, then energy would be getting lost and the AMR function would not be accurate.

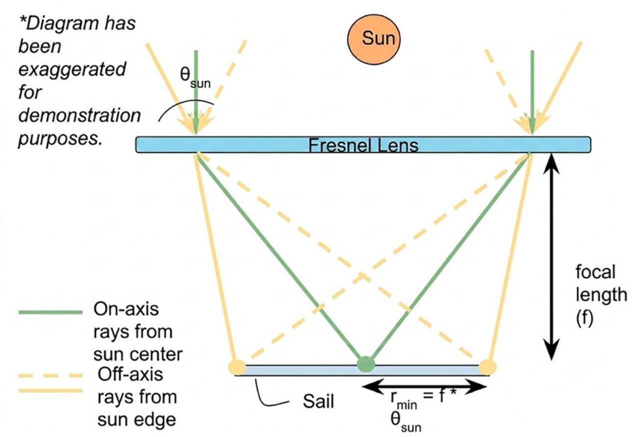

***Figure 5.* Ray diagram representing Sun Image Projection.** *The sun cannot be considered a point source, and thus the image of the entire sun must fit on the sail for it to make maximum use of radiation pressure from the sun.*

To be specific, assuming the sun is approximately 1 AU (astronomical unit, or about 1.49×10^{11}) away, the angular diameter $\theta_{sun} = \arctan\frac{D_{sun}}{d_{AU}} = \arctan\frac{1,392,000\ km}{149,600,000\ km} \approx 0.533° = 0.0093rad$

This means that the minimum diameter required is $f \cdot tan(\theta_{sun}) \approx f \cdot \theta_{sun}$ using the small angle approximation (since θ_{sun} is very small). This means that the minimum radius of the solar sail required is $r_{min} = \frac{f \cdot \theta_{sun}}{2}$. However, f also depends on r (as $f = R / (n - 1)$ by the thin lens approximation and R depends on r as described by Eqn. 2), meaning the equation is implicit. We can solve by substituting the equation for f in terms of R and substituting the equation of R in terms of r, which leads to the following equation: $r = \frac{\theta_{sun}}{2(n-1)} \cdot \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ where a, b and c refer to the terms in the equation of R(r) above.

Fresnel Lens Optimization via Desmos

We must also decide on a general form for the formula of a Fresnel lens in a 2D plane, where in 3D it would be concentric circles created by rotating it over the y-axis. The formula would be, by defining the function of the Fresnel lens as F(x), $F(x) = h - mod\left(R\left(1 - \sqrt{1 - \left(\frac{x}{R}\right)^2}\right), h\right)$. This addition of h and turning the function negative makes the function positive or 0 for all x, as well as having the integral under the curve being the correct area.

Meanwhile, we can also write the equation to be optimized. The equation for AMR can be derived by $\frac{SA}{mass} = \frac{\pi R^2}{d_w(volume\ of\ Fresnel\ lens) + d_r(volume\ of\ solar\ sail)}$

The volume of the Fresnel lens from the region (r,R) can be found by integrating: $2\pi \int_r^R (x \cdot F(x))dx$. Thus, the $\frac{SA}{mass}$ ratio is:

$$\frac{\pi R^2}{d_w \cdot 2\pi \int_r^R (x \cdot F(x))dx + d_r \cdot h \cdot \pi r^2} = \frac{R^2}{2d_w \int_r^R (x \cdot F(x))dx + d_r h r^2}.$$

Considering a similar solar sail of a circular design with similar boom length would lead to a more accurate comparison between a sail with and without a Fresnel lens. This design has an AMR of 340 m^2/kg without considering factors such as coatings and reinforcements. Thus, the equation becomes: $\frac{R^2}{2d_w \int_r^R (x \cdot F(x))dx + d_r h r^2}$ > 340 m^2/kg. Then, by plugging in the equation to the online calculator Desmos, an approximate maximum was calculated for the AMR, which was around 975 m^2/kg. This value was an early estimate from a coarse Desmos sweep that did not yet account for the rapid oscillation of the lens profile; the Python optimization below, which applies the averaging theorem and the sun-image radius constraint, supersedes it, and the paper adopts 680.27 m^2/kg as the optimized result.

The usage of the python model allowed further insights discussed in the Results section on the relation between AMR and the radius of the sail. When trying to calculate the optimal radius of the sail using python, there were inaccuracies due to lack of computation capacities that did not reflect results from Desmos despite identical parameters. These methods, in order, are: using python's max function, using small increments, using summation instead of integration to reduce computation complexity. Thus, the optimal AMR derived via the python code was largely ignored.

Fresnel Lens Optimization via Python

The difficulty in approximation was later found to be due to jumps caused by the modulo function of the Fresnel lens, under which numerical integration by programs like Python and Desmos fails due to the continuous jump discontinuities. Thus, an approximation method is needed to convert the Fresnel lens function into an integrable expression. This sawtooth-like pattern (Figure 1) with each "tooth" occurs whenever the term $\left(R\left(1 - \sqrt{1 - \left(\frac{x}{R}\right)^2}\right)\right)$ exceeds h. Thus, the number of teeth, N, can be defined as $\frac{\left(R\left(1 - \sqrt{1 - \left(\frac{x}{R}\right)^2}\right)\right)}{h}$.

Expanding using the Taylor Series for $\sqrt{1 - \left(\frac{x}{R}\right)^2}$, gives $1 - \frac{x^2}{2R^2} + O\left(\left(\frac{x}{r}\right)^4\right)$ (where O represents the error). The O term approximates to 0 as w≪R, so the term $1 - \sqrt{1 - \left(\frac{x}{R}\right)^2} \approx 1 - (1 - \frac{x^2}{2R^2}) \approx \frac{x^2}{2R^2}$. Since the function oscillates rapidly (as R ~ 0.1m and h = 2.115 × 10^{-6} m, meaning R/h ~ 5 × 10^4), height across the lens can be approximated to h/2 over a large interval by using the averaging theorem for rapidly oscillating functions (11).

Thus, $F(x) = h - mod\left(R\left(1 - \sqrt{1 - \left(\frac{x}{R}\right)^2}\right), h\right) \approx h - \frac{h}{2} = \frac{h}{2}$.

Using these approximations, the volume of the lens becomes $2\pi\int_r^R (x \cdot \frac{h}{2})dx = \pi h \cdot \frac{R^2 - r^2}{2}$. Thus, $\frac{SA}{mass} = \frac{R^2}{\frac{h}{2}[d_w(R^2 - r^2) + 2d_r r^2]}$. Using the equation for R derived above, $R(r) = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ where $a = \frac{1}{(n-1)^2} + \frac{1}{4}$, $b = -r/2$, and $c = \frac{r^2}{4} - \left(\frac{28 - 6r}{6}\right)^2$, the AMR can be optimized in terms of the variable r to find the optimal radius and highest AMR possible.

An optimization algorithm in Python was created to find the optimal radius of the solar sail (r) that leads to the maximum AMR. Limitations considering minimum radius possible were also implemented, leading to an AMR of 680.27 m^2/kg for Mylar at a solar sail radius of 0.020484 meters and an AMR 665.91 m^2/kg at a solar radius of 0.020190 meters. Further discussion of AMR and solar sails considerations will use the 680.27 m^2/kg value from Mylar as this is the greater (and thus more optimal) value.

Considering Losses

Another aspect to test the Fresnel lens on is how focused the light will be as that affects the transfer of momentum. The software used to simulate this situation was Ansys Lumerical FDTD which is used for modeling light interactions. This software was used to model plane wave interactions with a Fresnel lens (modeled using an equation of the Fresnel lens). To figure out how light would optimally push the sail forward, the optimal wavelength of light was used for the simulation, which is in the 705-750 nm range (12). Through this, the simulation was run for both materials, leading to the production of the Fresnel lens and the intensity of the light (how focused the light is; Figure 7). The figures were generated with .lsp files.

The equation inputted into Lumerical FDTD followed the general equation of a Fresnel lens provided by Ansys Lumerical: mod $(R * (1 - \sqrt{(1 - (u * R - 1)^2)}), h)$ (13), where the R was plugged in was the optimal value of R for each respective material. Meanwhile, the material for each simulation was changed as well to have the correct refractive index. However, due to the calculations needed to simulate a lens with dimensions meters in length that would take up an exceedingly high amount of RAM, only part of the lens could be simulated.

The effective AMR can be defined as the AMR considering losses. This can be mathematically represented as $\eta_{total} \times AMR$, where η_{total} is the factor considering all losses. The losses that will be considered in this section are lens transmittance, diffraction efficiency, momentum transfer, along with a thermal and stress check to ensure that these factors do not have any effect on the AMR. When comparing this to the ACS-3 solar sail, some of these losses will need to be considered to keep the comparison accurate. Mylar will be the only material considered for these simulations as it was determined to be the more optimal of the two materials. To find the transmittance of the lens, the data extracted from the FDTD simulation can be used using a broadband Gaussian source that mimics the sun. The transmittance of the lens η_T was determined to be *0.9078* for Mylar. However, any photons reflected off the lens also contribute to the propulsion off the craft, and only the energy that is absorbed is lost. Thus, the true "effective" transmittance factor is 0.9992 (Figure 6A). For the ACS-3 solar sail, a Fresnel lens is not used, so this loss does not apply.

The reflectivity of the sail can also be determined by using parameters regarding the sail material and thickness. The ACS-3 Sail is made of an Al/PEN/Cr stack (14) to maximize reflectivity while reducing the thermal stress. The respective thickness of each part of the stack is 100 nm, 2.5 um, and 15 nm. Using the transfer matrix method for this stack can verify and account for the complex refractive index and the optical phase thickness, and then weighted across the AM0 solar irradiance. Thus, η_R is 0.9225 (Figure 6B). However, since reflectivity is a loss both the Fresnel sail and the ACS-3 have, it will not be factored in when comparing their effective AMR.

Next, the diffraction efficiency can also be derived through RCWA analysis by expanding fields in diffraction orders then solving the Maxwell equations for each. This analysis helps determine what portion of the light passing through the lens truly ends up hitting the sail, as only the +1 (first) order hits the sail. Thus, η_D is 0.5608 (Figure 6C), making diffraction the most significant loss. It can also be seen that the reflectance determined using RCWA analysis of the sail is similar to that determined using FDTD, showing consistency across the results.

Then, an analysis of off-axis rays can be done to see the effect of non-parallel rays on the performance of the sail. The sail itself was analyzed for its efficiency using the TMM method mentioned above while the lens was simulated using FDTD. Using and testing sample data points at 0, 15, 30, 42 and 55 degrees as the angles of incidence, it can be seen that the focusing of the lens on the sail (the diffraction efficiency) drops sharply after 15 degrees and does not focus at all after ~42 degrees (Figure 6D), proving that the sail is quite sensitive to changes in the angle.

Finally, a thermal and stress check can be performed on the sail to ensure that it is capable of operating within the extreme temperatures and radiation pressures that it would be subject to in space. Assuming that the sail is operating at 1 AU near the Earth, it reaches radiative equilibrium when the absorbed solar flux equals the power radiated from both faces (thus the temperature will remain the same). This implies that $\alpha I_\odot = 2\varepsilon\sigma T_{eq}^4$ (using the Stefan-Boltzmann Law). The solar absorbance α of Mylar is ~0.14 (15), the IR emittance of Mylar is ~0.28 (16), and solar irradiance $I_\odot$ is 1361 W m⁻² at 1AU.

Thus, $T_{eq} = \left(\frac{\alpha I_\odot}{2\varepsilon\sigma}\right)^{\frac{1}{4}} = \left(\frac{0.14 \times 1361}{2 \times 0.28 \times 5.67 \times 10^{-8}}\right)^{\frac{1}{4}} = 278K$

which represents a safe operating temperature for Mylar. Structural integrity and stability under radiation pressure can also be calculated. The sail can be modeled as a thin circular membrane of radius $R_* = 2.914m$ with thickness $h = 2.115 \times 10^{-6}$. Thus, the radiation pressure is $P_{rad} = \frac{2I_\odot}{c} = \frac{2 \times 1361}{2.998 \times 10^8} = 9.07 \times 10^{-6}\ Pa$. For a laterally loaded circular membrane the peak stress occurs at the centre and is given by $\sigma_{max} = \frac{P_{rad} R_*^2}{4h^2} = \frac{9.07 \times 10^{-6} \times (2.914)^2}{4 \times (2.115 \times 10^{-6})^2} = 4.31\ MPa.$

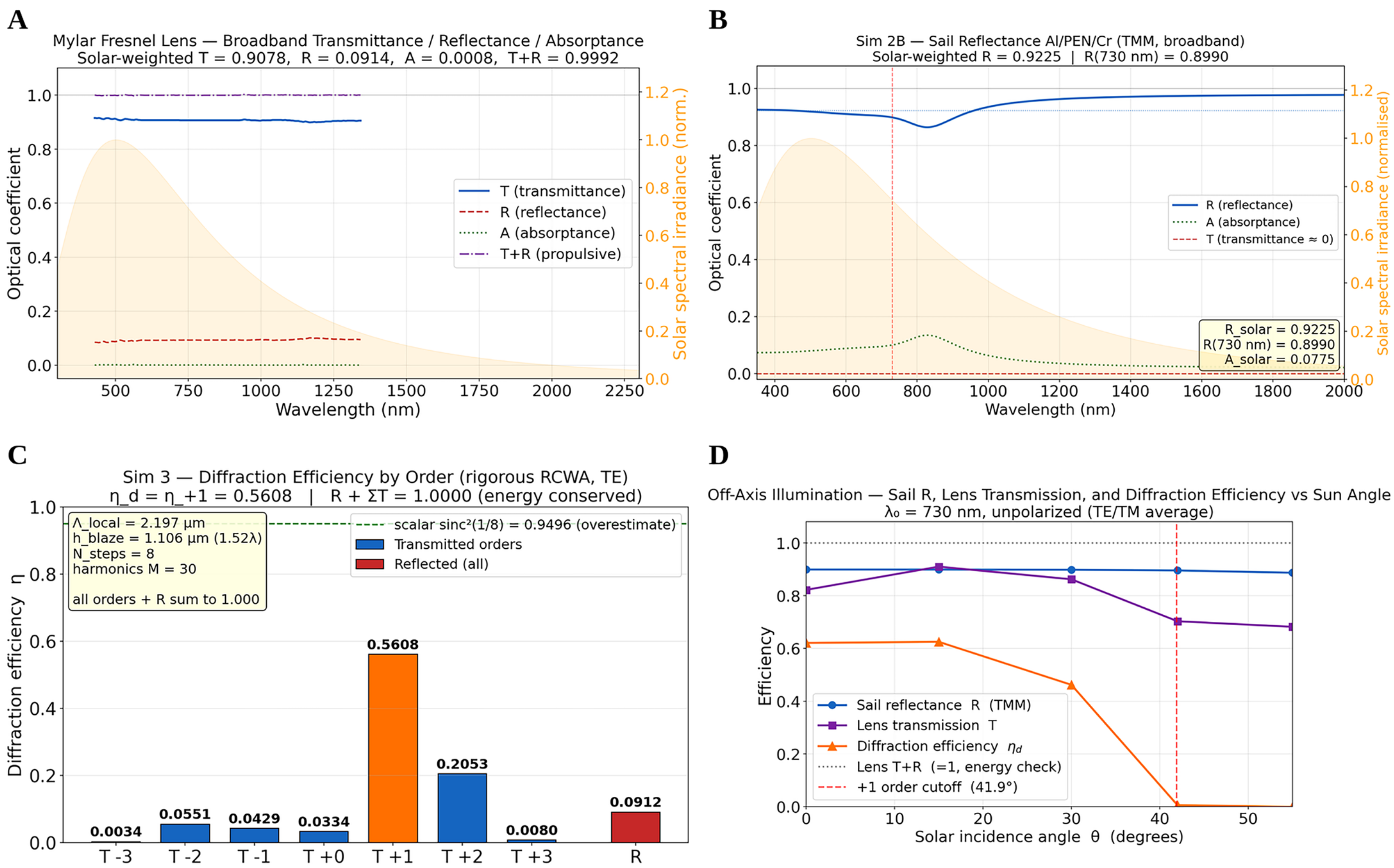

Figure 6. Optical simulation results for the Fresnellens solar sail system. *(A) Broadband transmittance, reflectance, and absorptance of the Mylar Fresnel lens (FDTD): solar-weighted T = 0.9078, R = 0.0914, A = 0.0008, giving an effective propulsive factor T + R = 0.9992. (B) Broadband reflectance of the Al/PEN/Cr sail stack (TMM): solar-weighted R = 0.9225. (C) Diffraction efficiency by transmitted order (RCWA): the +1 order, which directs light onto the sail, carries a diffraction efficiency of 0.5608. (D) Off-axis performance: sail reflectance, lens transmission, and diffraction efficiency versus solar incidence angle, with the +1 order cutoff at approximately 41.9 degrees. Shaded regions in (A) and (B) show the normalized AM0 solar spectral irradiance.*

The tensile strength of Mylar is 234.4 MPa (17), meaning that the stress the Mylar will undergo is well under its tensile limit. Thus, the lens structure is stable under radiation from the sun.

RESULTS

In the mathematical approach to finding the optimal Fresnel lens, for the material of Mylar, the optimal r was calculated to be 0.020484 meters, granting an AMR of about 680.27 m^2/kg. Multiplying all the losses ($\eta_R \times \eta_T \times \eta_D = 0.9225 \times 0.9992 \times 0.5608$) leads to an effective AMR of $0.52 \times 680.27\ m^2/kg$, yielding an effective AMR of about 351.65 m^2/kg. When comparing the sail to the ACS-3, the reflectance loss is present in both, so the comparative AMR is 381.19 m^2/kg. Measured against the ACS-3 theoretical AMR of ~340 on this same loss-adjusted basis (reflectance excluded from both designs), this is a 1.12x improvement. For the material of Kapton, the optimal r is at 0.020190 meters, granting an AMR of about 665.91 m^2/kg. The effective AMR and comparative AMR were not calculated for Kapton as Mylar was determined to be the more optimal result. The Mylar result, when considered under idealized thin-lens limits, nearly doubles the theoretical AMR of the ACS-3 solar sail (a 2.0x improvement, or 1.12x once losses are applied to both designs on the same basis). However, current manufacturing techniques limit this value greatly.

At the higher values of thickness, a clear trend of lower AMR with higher radius emerges, with the theoretical AMR peaking around 680.27 m^2/kg at an approximate r of 0.066 m for Mylar and approximately 0.106 m for Kapton from the graph (note: the refined analytical optimization in the Methods, which includes the averaging theorem and sun-image constraint, yields a more precise optimum of r = 0.020484 m for Mylar). This max value is around 15 m^2/kg considering Fresnel lens manufacturing limits that limit the thickness to 0.13mm (18), essentially deeming Fresnel lenses ineffective. However, at lower values of thickness, this trend is harder to discern and leads to theoretical values that exceed an AMR of 900. There are some spikes in the graph likely due to distortion at the edges of the Fresnel lens, a pattern that warrants further investigation.

In the end, these findings show that this unique approach of utilizing Fresnel lenses in solar sails could boost the sail's efficiency, though significant practical challenges remain as discussed below.

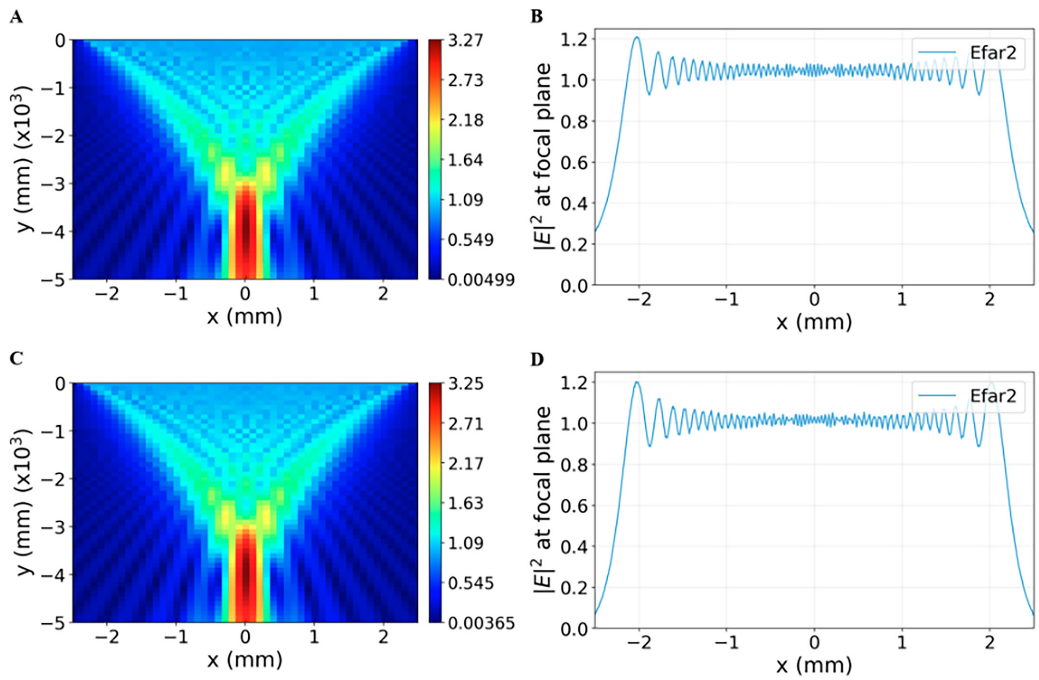

Figure 7. ***Lumerical FDTD simulation results.*** *(A) Kapton: aberration of light output at the focal plane; x-axis is lateral distance from lens center (mm), y-axis is electric field amplitude. (B) Kapton: intensity distribution from the lens surface; x-axis is lateral distance from lens center (mm), y-axis is axial distance from lens surface (µm). (C) Mylar: aberration of light output, axes same as (A). (D) Mylar: intensity distribution, axes same as (B). In both materials, focal spread is on the order of a few millimeters, negligible relative to meter-scale sail dimensions.*

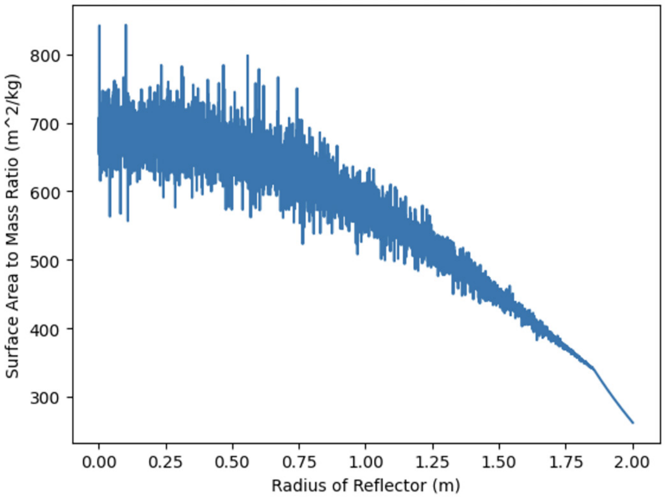

Figure 8. ***AMR Optimization Graph.*** *This graph shows at which radius the surface area to mass ratio of the sail becomes optimized with the usage of a Fresnel lens made of Mylar (x-axis is sail radius r (m), y-axis is AMR (m²/kg)). The graphical peak appears between r = 0.05-0.1 m, though the refined analytical optimization yields r = 0.020484 m (see Results).*

DISCUSSION

Several factors affected the scope and conclusions of this study and merit consideration. This paper looked mainly into the conceptual designs of incorporating Fresnel lenses into solar sails rather than deeply diving into the material science aspect of creating a solar sail, thus likely causing some faulty assumptions to make the results not match with reality. Some examples include the assumption that it is possible to make a Fresnel lens as thin as 2.115 microns but still be able to operate in space and that the focus of the Fresnel lens will not burn.

Second, this paper was unable to physically test this design due to costs and lack of access to cutting-edge materials. Therefore, this paper is entirely theoretical in nature, and physical validation remains necessary before any engineering conclusions can be drawn.

Meanwhile, shifting toward the theoretical side, in the mathematical calculations, the maximum thickness of the Fresnel lens was made the same thickness as the solar sail, which is likely not possible to manufacture with current technology due to the precision required to make thin concentric circles. Even if the technology for Fresnel lens thickness improves and decreases the minimum thickness for Fresnel lenses, it is likely that similar advances would also reduce solar sail thickness. Looking at current Fresnel lens technological capabilities, the minimum commercially available Fresnel lens thickness is 0.13 mm (18), more than 60 times the 2.115 microns assumed in this study. At this thickness, the area-mass ratio drops to approximately 15 m^2/kg, which is worse than the AMR of a solar sail without any Fresnel lenses at all. This is an important fact to consider as it makes the feasibility of using Fresnel lenses in solar sails an impractical design choice and fails to support its original purpose of making Fresnel lenses a useful technology in increasing the AMR of a solar sail.

The results of the optimization are also sensitive to the parameters inputted. Thickness has the greatest effect on AMR. The AMR formula approximates as SA/mass $\propto$ 1/h, so the ~62x increase from the assumed 2.115 µm to the minimum manufacturable 0.13 mm (18) lowers the AMR by about 98%, from ~680.27 to ~15 m²/kg. Figure 8 implicitly shows this sensitivity through its multiple thickness curves, but the result is that this maximum is only achievable through fabrication that is yet to exist. Refractive index also affects the result through the focal length. $f = R/(n-1)$, so a variation in n directly shifts the boom geometry and the feasible range of r. For Mylar at n = 1.66, a ±0.01 variation, which is realistic across different material grades or wavelengths within the 705–750 nm simulation range, changes $n-1$ by around 1.5% and propagates into R(r) and the reported optimum. Density affects AMR through the same mass denominator as thickness. A denser material means more mass for the same geometry, which directly reduces the ratio. Commercial Mylar grades vary by a few percent depending on processing conditions, and any coatings or structural reinforcement, both of which this paper explicitly excludes, would further bring AMR down by adding mass. The reported optimum of ~ 680.27 m²/kg should therefore be understood as a theoretical upper bound that assumes exact values for all three parameters simultaneously, rather than a robust design target. Batch testing of all realistic material tolerances would be required to establish a proper and meaningful operating range and provide confidence in the stability of the reported optimum value.

Taken together, the three main assumptions in this study present feasibility challenges. Although the thin-lens approximation is mathematically convenient, it assumes that the lens thickness is negligible compared to the focal length. At the scale of the lens with R being around 0.1 m, this assumption breaks down: the groove edge effects and the phase errors become meaningful, meaning real-life focal properties would begin to diverge from the predictions here. Separately, actually manufacturing a functional Fresnel lens with a thickness of 2.115 µm that matches the sail material surpasses the capabilities of any fabrication method for such a system. Finally, the heating at the focal point of the sail due to the concentration of sunlight from the lens leads to localized heating that could deform the sail or decrease reflectivity over time, despite the Stefan-Boltzmann analysis in the Methods that suggests the system remains within these limits. None of these limitations alone is necessarily detrimental to the concept, but addressing all of them simultaneously would need advances in fabrication, optical engineering, and thermal management, none of which are reasonable in the near future.

Another major drawback to note is the size of the solar sail. In the optimization, the optimal radius was reported to be ~0.02 meters, which is extremely small and is not feasible considering that a Fresnel lens of a proportionally bigger size would need to have its own deployment and storage system along with the sail. The extremely small size would also force the sail to function solely as a reflector for the Fresnel lens and not as a sail itself. This conclusion arises from the fact that the lens itself has a better AMR than the sail, and thus maximizing the amount of lens used improves the AMR. And considering the thickness limitations mentioned above, this would not be a logical design choice. Furthermore, if $r \approx 0.02$ m, the amount of boom supporting the sail is only $6 \times 0.02 = 0.12$ m of boom length, meaning the remaining ~27.88 m must come from the Fresnel lens support booms (by Eqn. 1). This implies an enormous lens-to-sail distance relative to the sail size, which is structurally unstable and presents further challenges related to the deployment and usage of the system.

On the simulation side, due to massive RAM requirements, the entire Fresnel lens could not be simulated, but rather only the center of it, which admittedly may not be the most helpful part to simulate since the middle of the Fresnel lens would be removed in this design. Further, because Lumerical uses a meshing process to calculate approximations at certain intervals in space, it is not designed to work effectively with large dimensions, which may be a cause of differences between the results and reality. These could lead to a decrease in the accuracy of the AMR due to its effect on the calculated losses on the solar sail. Furthermore, other factors such as full scale optical behavior and photon pressure transfer could not be modeled, which leads to further neglect of factors that could have a significant effect on the AMR (such as the losses that were considered previously).

Aside from research and experimental flaws, there are also some aspects of solar sails that this paper neglected but could be further explored and improved in future research.

One aspect of the experiment that could be improved on and mitigated in the future research is the aberration caused by the Fresnel lens. Because this paper only looked at optimizing AMR which is not the only determining factor for the efficacy of a solar sail, aberration was not factored in as a component in the optimization, so future research can make better models for Fresnel lens optimization with that in mind.

This paper also disregarded the factor of compactness in deciding the effectiveness of a solar sail. Future research that takes into account the compatibility of a Fresnel lens solar sail with compactness could make the design of the Fresnel lenses more feasible to launch.

Also, with Fresnel lenses, the solar sail can be steered forward, but no experiment or research has been conducted to see if changing the Fresnel lens angle to steer the solar sail would be practical. A large incentive and advantage for using Fresnel lenses lies in their capability to change angle while the sail is less able to, thus inspiring past conceptual research into using Fresnel lenses for maneuverability (19). By adding maneuverability to the optimization in the study, future research could make this paper's design better.

CONCLUSION

This study shows a theoretical analysis of the usage of Fresnel lenses in circular solar sails to optimize AMR. Under ideal conditions that assume a lens thickness of 2.115 µm, an AMR of around 680.27 m^2/kg is achievable, nearly doubling the theoretical ACS-3 value of ~340 m^2/kg (a 2.0x improvement, narrowing to 1.12x once optical losses are applied to both designs on the same basis). However, when the minimum manufacturable thickness is accounted for (0.13 mm (18)), the AMR drops to approximately 15 m^2/kg, below even a standard solar sail without any Fresnel lens. The approximately 680.27 m^2/kg value is a very idealized upper value that assumes exact values for optical, fabrication, and material parameters, not a currently feasible engineering target. Realizing these theoretical gains would require advances in fabrication precision, thermal management, and optical engineering at a scale that remains beyond near-term capability.

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CONFLICTS OF INTEREST

The authors declare no conflicts of interest related to this work

DATA AND CODE AVAILABILITY

The optimization and simulation work underlying this study is openly available. The boom-geometry and area-to-mass-ratio relationships were explored with the Desmos graphing calculator (https://www.desmos.com/calculator/45c028391d; https://www.desmos.com/calculator/9bcbea47a4). The Fresnel-lens area-to-mass-ratio optimization was implemented in Python via Google Colab (https://colab.research.google.com/drive/1F1p0p_-Bk8F9qlVnxEEeoSlKh-yq1gbj; https://colab.research.google.com/drive/1TbtlFZ3viUUdTk5TtwfkNFnLFfI4vHp5).

REFERENCES

1. NASA's advanced composite solar sail system (Internet). The Planetary Society; 2024 Available from: https://www.planetary.org/space-missions/acs3 (accessed on 2026-01-02)
2. Ren Z, Li C, Wu K, Wang Z, Wang H, Yan P. Design, modeling and experimental investigation of a novel solar sail with high area-to-mass ratios for efficient solar sailing. *Chin J Aeronaut*. 2024. doi:10.1016/j.cja.2024.01.033
3. Han JW, Kong YJ, Park TK, Ohn YH. The shape of Fresnel lens. *J Korean Ophthalmol Soc*. 2015; 56 (12): 1874–9. doi:10.3341/jkos.2015.56.12.1874
4. Park TY, Jung JY, Oh HU. Experimental investigation on the feasibility of using a Fresnel lens as a solar-energy collection system for enhancing on-orbit power generation performance. *Int J Aerosp Eng*. 2017; 2017: 1435036. doi:10.1155/2017/1435036
5. Wilkerson G, Huegele V. The optical design of a system using a Fresnel lens that gathers light for a solar concentrator and that feeds into solar alignment optics (Internet). NASA; n.d. Available from: https://ntrs.nasa.gov/api/citations/19980201003/downloads/19980201003.pdf (accessed on 2026-01-02)
6. Johnson K. Fresnel lens condenser for a solar sail (preprint) (Internet). viXra; 2017 Available from: https://vixra.org/pdf/1710.0349v1.pdf (accessed on 2026-01-02)
7. MatWeb material property data (Internet). Blacksburg (VA): MatWeb; Available from: https://www.matweb.com/index.aspx (accessed on 2026-01-02)
8. Davoyan AR, Munday JN, Tabiryan N, Swartzlander GA, Johnson L. Photonic materials for interstellar solar sailing. *Optica*. 2021; 8 (5): 722–34. doi:10.1364/optica.417007
9. Wilkie K, Fernandez J, Heiligers J, Carzana L. Advanced composite solar sail system (ACS3) mission update (Internet). NASA; 2023 Available from: https://ntrs.nasa.gov/api/citations/20230008378/downloads/Wilkie_ACS3_mission_update_ISSS_2023_20230530_rev_a.pdf (accessed on 2026-01-02)
10. Kezerashvili VY, Kezerashvili RY. Theoretical approach to circular solar sail deployment. *Adv Space Res*. 2024; 73 (9): 4731–41. doi:10.1016/j.asr.2024.02.002
11. Sandrakov GV. Averaging principles for equations with rapidly oscillating coefficients. *Math USSR Sb*. 1991; 68 (2): 503–53. https://doi.org/10.1070/SM1991v068n02ABEH002111
12. Ma D, Munday JN. Measurement of wavelength-dependent radiation pressure from photon reflection and absorption due to thin film interference. *Sci Rep*. 2018; 8: 34381. doi:10.1038/s41598-018-34381-z
13. Fresnel lens (Internet). Ansys Optics; 2025 Available from: https://optics.ansys.com/hc/en-us/articles/360042260073-Fresnel-lens (accessed on 2026-01-02)
14. Kang JH, Gordon KL, Bryant RG, Stohlman OR, *et al*. Durability characterization of mechanical interfaces in solar sail membrane structures. *Adv Space Res*. 2021; 67: 2643–57. doi:10.1016/j.asr.2020.08.015
15. Solar reflectance, transmittance and absorptance of common materials (Internet). Golden (CO): Solar Energy Research Institute. Report No.: SERI/TP-334-457. Available from: https://docs.nrel.gov/docs/legosti/old/457.pdf (accessed on 2026-06-19)
16. Solar absorptance and thermal emittance of some common spacecraft materials (Internet). Fort Belvoir (VA): Defense Technical Information Center. Report No.: ADA305864. Available from: https://apps.dtic.mil/sti/tr/pdf/ADA305864.pdf (accessed on 2026-06-19)
17. Mylar polyester film: physical and thermal properties

(datasheet) (Internet). Laminated Plastics; Available from: https://laminatedplastics.com/mylar.pdf (accessed on 2026-06-19)

18. Fresnel lenses for infrared wavelengths (Internet). Fresnel Technologies; n.d. Available from: https://www.fresneltech.com/hubfs/Spec%20Sheets/POLYIR%20Brochure.pdf (accessed on 2026-01-02)
19. Quarta AA, Mengali G. Solar sail orbit raising with electro-optically controlled diffractive film. *Appl Sci*. 2023; 13 (12): 7078. doi:10.3390/app13127078