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Dynamic Multi-Asset Portfolio Optimization: Evaluating Risk-Return Tradeoffs Under Time-Varying Volatility

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ABSTRACT

Prior literature on the low-volatility anomaly suggests that portfolios composed of lower-volatility assets often achieve superior risk-adjusted returns compared to their higher-volatility counterparts over long investment horizons. This phenomenon challenges the traditional risk-return trade-off implied by classical asset pricing models, which associate higher risk with higher expected returns. As a result, low-volatility strategies have gained attention for their ability to deliver more efficient return profiles with reduced downside risk. This study investigates this claim by comparing alternative portfolio allocation strategies in optimizing the risk-return tradeoff over the 2021-2025 period. Using volatility forecasts generated through a GARCH (1,1) model and evaluating two allocation frameworks - volatility targeting and Sharpe ratio-constrained optimization, we examine performance across varying market conditions. The findings indicate that Sharpe ratio-constrained optimization produces higher returns during strong market recoveries; however, it is vulnerable to significant drawdowns and elevated tail risk during market downturns. In contrast, the volatility-targeting strategy demonstrates greater stability, lower maximum drawdowns, and more consistent risk-adjusted performance in adverse market environments. Overall, the results suggest that dynamic rebalancing and active risk management are critical determinants of long-term portfolio performance. High-volatility assets, such as Bitcoin, may enhance returns, but only when their exposure is carefully managed within a diversified portfolio framework.

Keywords: Dynamic Portfolio Optimization; Mean-Variance Optimization (MVO); GARCH Volatility Modeling; Risk–Return Tradeoff; Multi-Asset Diversification; Sharpe Ratio; Treynor Ratio; Volatility Targeting

INTRODUCTION

Modern portfolio theory originates from Markowitz's (1952) mean-variance optimization (MVO) framework (1), which assumes normally distributed asset returns and stable risk parameters, including volatility and cross-asset correlations. Under this framework, investors seek to maximize expected returns for a given level of risk, measured by variance. However, extensive empirical evidence demonstrates that financial markets frequently violate these assumptions due to volatility clustering, fat-tailed return distributions, and time-varying risk dynamics (2-5). As a result, traditional MVO may be insufficient in capturing real-world portfolio behavior, particularly during periods of market stress.

A significant challenge to the assumption of constant volatility emerges from research on Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized ARCH (GARCH) models, which demonstrate that volatility is conditional and evolves over time (6).

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Empirical findings suggest that portfolios constructed using GARCH-based volatility forecasts generate more stable and realistic asset weights than portfolios relying solely on historical variance estimates (6). Furthermore, multivariate extensions such as Dynamic Conditional Correlation (DCC-GARCH) models have been shown to improve risk-adjusted returns during volatile market conditions. Related research employing stochastic volatility frameworks indicates that expected returns and covariances vary over time, implying that optimal portfolio allocations are inherently dynamic and sensitive to investment horizons.

Dynamic ass*et al*location frameworks further challenge traditional static portfolio construction by adapting to evolving market conditions and changing risk profiles over time. Prior studies demonstrate that incorporating time-varying inputs, such as volatility and expected returns, can lead to improved portfolio performance and more effective risk management across different market regimes. (6, 7). Studies demonstrate that portfolios adjusting to changing macroeconomic and financial conditions outperform static allocations, particularly when accounting for model uncertainty in returns and correlations (2, 3). Time-varying diversification behavior and adaptive rebalancing appear to play critical roles in long-term performance (5, 8). Collectively, this body of literature supports the proposition that dynamic portfolio optimization is superior to static mean–variance approaches in environments characterized by structural instability and evolving risk (1, 4).

In addition to methodological considerations, diversification across multiple asset classes has become increasingly important. Research employing stochastic spanning approaches finds that incorporating commodities, real estate, foreign exchange, and alternative investments can significantly enhance risk-adjusted performance relative to portfolios limited to traditional equities and fixed income. These findings underscore the importance of multi-asset diversification in optimizing the risk–return tradeoff.

Moreover, a substantial body of literature critiques variance as a sole measure of risk (2, 3). Variance assumes return normality, penalizes upside and downside volatility symmetrically, and inadequately captures tail risk and time-varying volatility - features that are shown in financial return series (2, 3, 6, 7). Alternative performance metrics such as the Sharpe ratio and Treynor ratio measure returns relative to total and systematic risk, respectively, offering complementary perspectives on portfolio efficiency.

Against this backdrop, this study investigates the extent to which dynamic, multi-asset portfolio diversification, including equities, fixed income, real estate, commodities, and alternative investments, can optimize the risk–return tradeoff. It further examines how incorporating time-varying volatility forecasts and risk-adjusted performance metrics alters portfolio outcomes relative to traditional mean–variance optimization (1). The central hypothesis is that portfolios composed of high-volatility asset classes can outperform portfolios composed of low-volatility asset classes when exposure is dynamically adjusted according to prevailing risk conditions. By integrating GARCH-based volatility modeling (6, 7) with Sharpe and Treynor ratio–informed optimization, this research seeks to contribute to the growing literature on adaptive risk management and dynamic portfolio construction.

METHODS AND MATERIALS

Financial data used in this study was obtained from Yahoo Finance (yfinance API) using daily adjusted closing prices for the period January 2021 through December 2025. Returns were calculated on a daily basis and annualized where appropriate for this study. Data for this study were analyzed using a combination of risk-adjusted performance metrics, volatility forecasting models, and portfolio optimization techniques. Specifically, we employed the Sharpe ratio to measure risk-adjusted returns and the Treynor ratio to assess returns per unit of systematic risk (beta). Time-varying volatility was estimated using a GARCH (1,1) model, which provides short-term volatility forecasts and conditional volatility modeling. Tail risk was evaluated using Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) derived from both the return distributions and GARCH-based volatility estimates.

Portfolio allocations were evaluated using two distinct strategies: 1) S1 / volatility-targeted allocation - designed to equalize risk contributions across assets or target a specific annualized portfolio volatility. 2) S2 / constrained Sharpe/Treynor-optimized allocation - maximizes risk-adjusted performance while imposing explicit caps on portfolio volatility and CVaR.

Expected returns were estimated using historical mean returns over a rolling lookback window, while covariance matrices were derived from the sample covariance of returns. The Sharpe ratio–constrained optimization was formulated as a mean-variance optimization problem with constraints on volatility and

CVaR, incorporating long-only positions (i.e., excluding short selling) and portfolio weights bounded between 0 and 1. All allocations were back tested over the 2021-2025 period using a rolling, walk-forward methodology. The rolling back test employed a 252-day lookback window, with rebalancing conducted approximately every six months. Portfolio weights were then applied out-of-sample for the subsequent period. Transaction costs and trading frictions were not incorporated, reflecting an idealized implementation. Models were fitted on a lookback window, optimal allocations were determined under the relevant constraints, and performance was applied to the subsequent period. Realized portfolio performance, deviations between forecasted and realized volatility, VaR breaches, and maximum drawdowns (MaxDD) were recorded. This approach emphasizes a risk-first perspective, prioritizing stability and downside protection over pure profit maximization. By incorporating volatility forecasting, stress testing, and tail risk measures, the methodology captures both normal and adverse market conditions, providing a comprehensive evaluation of portfolio performance under dynamic market environments. All model estimations were performed in-sample within each rolling window, while the reported portfolio performance is based on out-of-sample results. The tables and figures are all included within the source code available on GitHub (9).

RESULTS AND DISCUSSION

Portfolio Allocations and Asset Characteristics

Table 1 provides a reference for the ticker symbols. Table 2 summarizes the optimal portfolio weights under two strategies: the Volatility Targeting Strategy (S1) and the Sharpe Ratio Constrained Optimization (S2) over 2022–2025. The asset universe included equities (SPY, QQQ), fixed income (TLT, LQD, HYG), and alternatives (GLD, VNQ, BTC-USD), representing a diversified multi-asset portfolio covering growth, income, and alternative exposures. SPY received a zero weight in the Sharpe-constrained optimization due to its lower risk-adjusted return relative to other assets under the imposed constraints. Equities provide core growth with varying volatility, fixed income manages interest rate and credit risk, and alternatives, such as gold, real estate, and Bitcoin, offer additional diversification and risk–return characteristics.

Under the Volatility Targeting Strategy (S1), allocations are inversely proportional to asset volatility. For example, BTC-USD, a highly volatile asset, received only a 0.0337 allocation, whereas lower-volatility assets such as HYG and LQD were allocated 0.2998 and 0.2227, respectively. This strategy prioritizes balancing

Table 2. *Optimal portfolio weights under the Volatility Targeting (S1) and Sharpe Ratio–Constrained Optimization (S2) strategies.*

Volatility targeted weights (pro-rated)	Constrained Sharpe weights
BTC-USD = 0.0337	BTC-USD = 0.0538
GLD = 0.0839	GLD = 0.4013
HYG = 0.2998	HYG = 0.5449
LQD = 0.2227	LQD = 0.0000
QQQ = 0.0677	QQQ = 0.0000
SPY = 0.0848	SPY = 0.0000
TLT = 0.1163	TLT = 0.0000
VNQ = 0.0911	VNQ = 0.0000

Table 1. *Asset tickers and corresponding financial instruments included in the portfolio analysis.*

Ticker	Name
BTC-USD	Bitcoin Priced in U.S. dollars (Cryptocurrency)
GLD	SPDR Gold Shares ETF (tracks the price of gold)
HYG	iShares iBoxx $ High Yield Corporate Bond ETF (junk/high-yield corporate bonds)
LQD	iShares iBoxx $ Investment Grade Corporate Bond ETF (high-quality corporate bonds)
QQQ	Invesco QQQ Trust (Nasdaq-100, tech-heavy equities)
SPY	SPDR S&P 500 ETF Trust (broad U.S. equity market)
TLT	iShares 20+ Year Treasury Bond ETF (long-term U.S. government bonds)
VNQ	Vanguard Real Estate ETF (U.S. real estate investment trusts, REITs)

risk contributions across assets rather than maximizing returns. Conversely, the Sharpe Ratio Constrained Optimization (S2) allocated 0.0538 to BTC-USD, 0.4013 to GLD, and 0.5449 to HYG, while assigning 0 to other assets. This allocation reflects the assets that optimized the risk-adjusted return, considering constraints on volatility and CVaR.

Risk-Adjusted Performance and Volatility Metrics

Table 3 shows individual asset characteristics, including annualized Sharpe ratios, volatility, and beta relative to the S&P 500. BTC-USD exhibited a high Sharpe ratio (0.7914) but extreme volatility (0.4931) and a high MaxDD (0.7663), indicating substantial tail risk. GLD and HYG demonstrated lower volatility and MaxDD while maintaining positive Sharpe ratios, justifying their higher allocations in both strategies. Negative Sharpe ratios for LQD (-0.2564) and TLT (-0.5102) explain their exclusion from the S2 allocation. Beta values indicate growth assets (QQQ, BTC-USD) are more sensitive to market movements, whereas safe-haven assets (GLD, TLT) provide portfolio stability.

GARCH (1,1) modeling provided conditional and forecasted volatilities (Table 4). BTC-USD, QQQ, and SPY maintained elevated forecasted volatility, consistent with volatility clustering, whereas bonds (LQD, HYG) exhibited low and stable forecasted volatilities. The GARCH (1,1) model was estimated using maximum likelihood, yielding parameters ($\omega = 0.45$, $\alpha = 0.08$, and $\beta = 0.88$) that reflect baseline variance, shock sensitivity, and volatility persistence. This stability explains the higher allocations to these assets under S1.

Tail Risk Analysis

Table 5 ranks assets by downside risk using VaR and CVaR. BTC-USD consistently exhibited the largest tail risk, while HYG and LQD showed minimal downside exposure. These findings illustrate the importance of incorporating tail risk measures alongside standard deviation when evaluating portfolio allocations.

Dynamic Back testing and Performance

These results suggest that S2 exhibits greater upside potential, but also increased fragility, particularly under changing market conditions. The rolling back tests, spanning multiple rebalance periods from 2021 to 2025, highlight how S2's performance tends to be more variable across different market regimes. This variability underscores the trade-off between return enhancement and stability when compared to the more consistent behavior observed in S1 (Figure 1, 2). Both strategies experienced losses during the 2022 downturn, but recovered during 2023–2025. The Volatility Targeting Strategy (S1) maintained lower and more stable annualized volatility (5–10%) and smaller MaxDD,

Table 4. *Conditional and forecasted volatility estimates derived from the GARCH (1,1) model for each asset.*

Ticker	Conditional Volatility	Forecasted Volatility
	Annual	Annual
BTC-USD	0.3611	0.3561
GLD	0.2344	0.2267
HYG	0.0378	0.0368
LQD	0.0468	0.0476
QQQ	0.1636	0.1598
SPY	0.1183	0.1155
TLT	0.0941	0.0936
VNQ	0.1447	0.1429

Table 3. *Risk and performance characteristics of individual assets: annualized Sharpe ratios, volatility, beta relative to the S&P 500, and maximum drawdown.*

Ticker	Sharpe (Annual)	Annual Volatility	Beta	Treynor	MaxDD	Total ROI
BTC-USD	0.7914	0.4931	1.1853	0.3292	-0.7663	7.085
GLD	0.686	0.1296	0.1047	0.8486	-0.2103	1.0691
QQQ	0.6304	0.1885	1.2466	0.0953	-0.3512	1.4023
VNQ	0.233	0.1585	0.7711	0.0479	-0.3448	0.3787
HYG	0.2112	0.0628	0.3333	0.0398	-0.1579	0.2542
LQD	-0.2564	0.0726	0.1822	-0.1021	-0.2495	-0.0089
TLT	-0.5102	0.1341	0.0513	-1.333	-0.4531	-0.3401

Table 5. *Comparison of daily and annual Value at Risk (VaR) and Conditional Value at Risk (CVaR) at the 95% confidence level across selected asset tickers.*

Ticker	VaR95_daily	CVaR95_daily	VaR95_ann_approx	CVaR95_ann_approx
HYG	0.006029	0.009667	0.095702	0.153454
LQD	0.007993	0.011147	0.126893	0.176947
GLD	0.012962	0.019656	0.205761	0.312024
SPY	0.013859	0.021768	0.220002	0.345558
TLT	0.014939	0.01988	0.237149	0.315581
VNQ	0.01674	0.024486	0.265746	0.388701
QQQ	0.01964	0.029158	0.311769	0.462867
BTC-USD	0.047135	0.070173	0.748242	1.113958

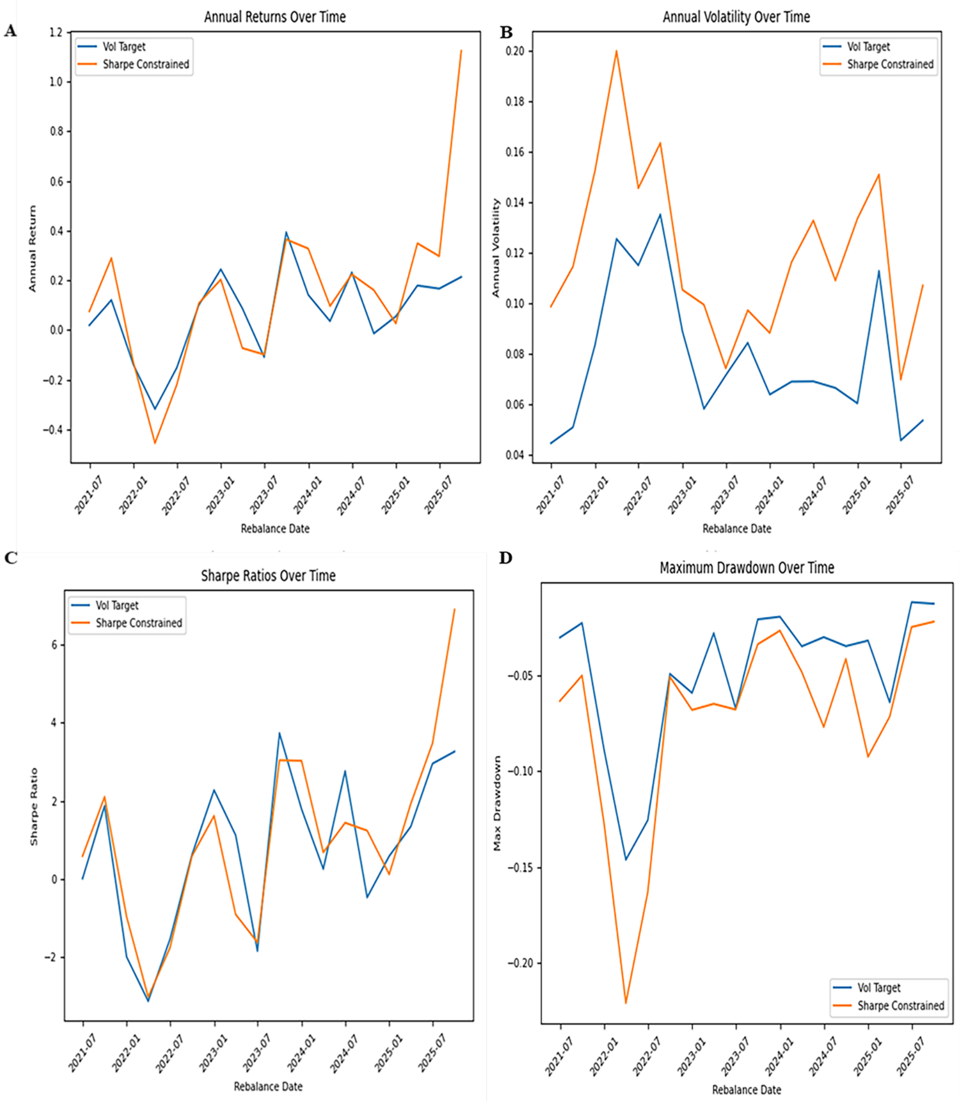

Figure 1. *Comparison of performance and risk metrics between the Volatility Targeted Portfolio and the Sharpe Ratio–Constrained Portfolio, based on the period from July 2021 to July 2025. (A) Portfolio returns over time. (B) Portfolio volatility profiles. (C) Risk-adjusted performance (Sharpe ratio). (D) Maximum drawdown comparison between strategies.*

resulting in smoother and more resilient portfolio performance. Sharpe-constrained allocations (S2) exhibited higher volatility (10–20%), larger drawdowns during downturns, and more negative Sharpe ratios, but achieved higher returns and Sharpe ratios during strong market recoveries. The 6-month rebalancing interval reflects a trade-off between responsiveness to changing market conditions and minimizing excessive portfolio turnover.

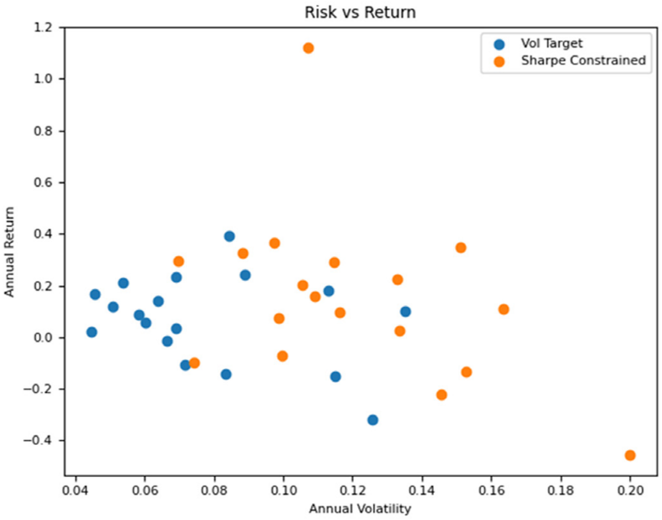

Figure 2. *Scatterplot of annual return versus annual volatility for both strategies, illustrating the risk-return tradeoff for each portfolio allocation strategy.*

Sharpe, Volatility, and Risk Management Observations

Figures 3-8 provide further insights into risk-adjusted performance and the behavior of the strategies over time. Figure 3 focuses on downside risk by presenting the maximum drawdown of the volatility-targeted portfolio, capturing peak-to-trough losses and thereby offering insight into tail risk and performance during periods of market stress. Figure 4 examines risk-adjusted performance through the Sharpe ratio, illustrating not only the efficiency of volatility conversion into returns but also its evolution over time. Complementing this, Figure 5 shows the annualized volatility profile, demonstrating the extent to which the portfolio maintains its targeted risk level and how realized volatility varies across the sample period. Figure 6 extends this analysis to returns by presenting annualized performance, highlighting how dynamic adjustments to maintain target volatility translate into overall return outcomes. Figure 7 synthesizes the risk-return relationship by illustrating the trade-off between return maximization and drawdown minimization, emphasizing the efficiency of downside risk control relative to achieved performance. Finally, Figure 8 compares cumulative returns of S1 and S2 over 2022 to 2026, providing a broader perspective on their relative growth trajectories and long-run performance differences.

S1 successfully controlled portfolio volatility within a 6–12% range, demonstrating the effectiveness of the

Figure 3. *Maximum drawdown of S1, illustrating peak-to-trough losses and providing a measure into tail risk and downside exposure during periods of stress. The y-axis represents the maximum loss, while the x-axis spans the period from July 2021 to July 2025.*

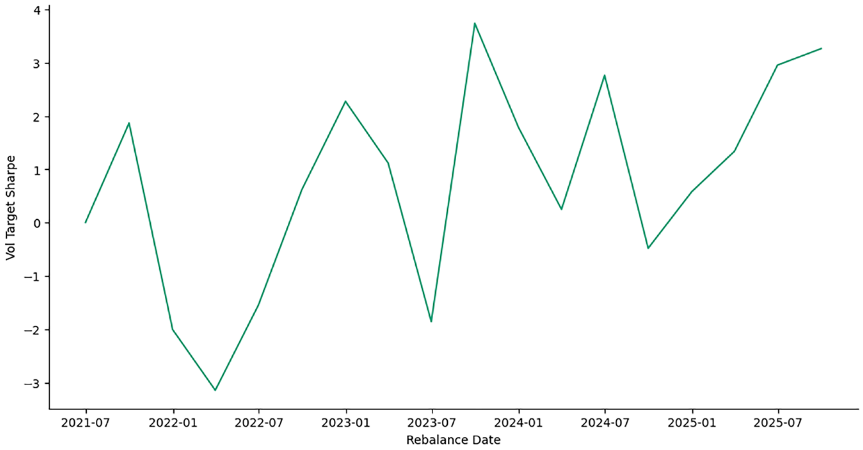

__Figure 4.__ Sharpe ratio of the S1, illustrating its risk-adjusted performance and the efficiency with which it converts volatility into returns. The y-axis represents the Sharpe ratio, while the x-axis spans the period from July 2021 to July 2025, highlighting how the metric evolves over time.

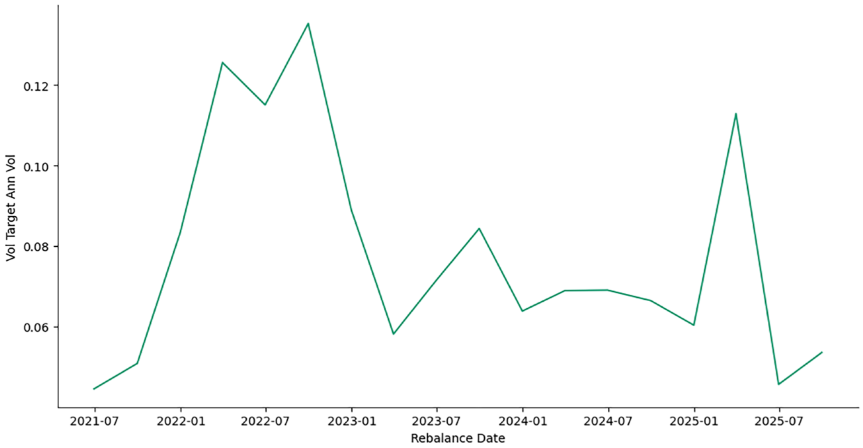

__Figure 5.__ Annualized volatility of the S1, illustrating how effectively the portfolio maintained its intended risk target over time. The y-axis represents annualized volatility, while the x-axis spans the period from July 2021 to July 2025, capturing the evolution of volatility throughout the sample period.

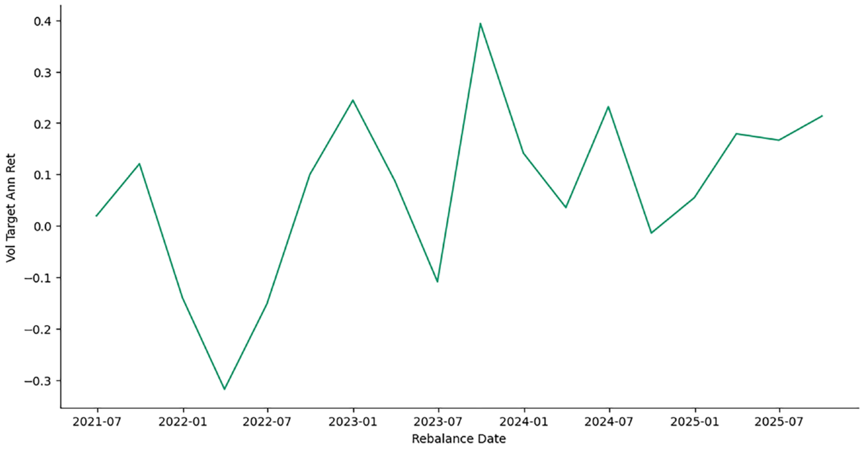

__Figure 6.__ Annualized return of S1, illustrating how adjustments in exposure to maintain the target volatility influence overall performance. The y-axis represents the annualized return, while the x-axis spans the period from July 2021 to July 2025, highlighting how returns evolve over time.

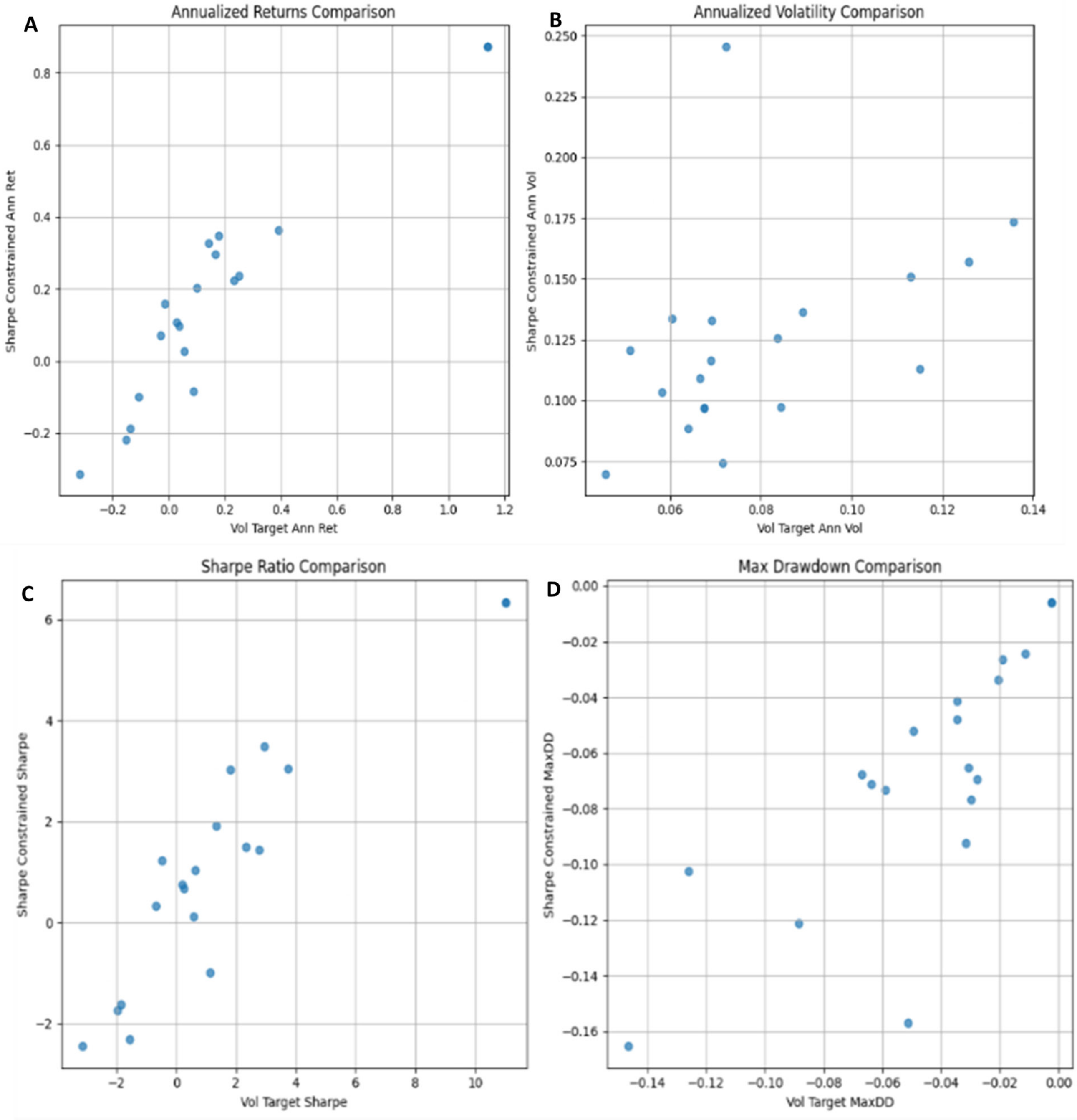

Figure 7. *S2 – Tradeoff between achieving higher returns and limiting drawdowns, highlighting the balance between return maximization and downside risk management. A: Relationship between portfolio volatility and risk-adjusted returns, illustrating whether higher volatility is associated with improved risk-adjusted performance. B: Relationship between portfolio risk and returns, highlighting whether higher-risk portfolios tend to deliver higher returns while also identifying potential outliers representing either inefficiency or exceptional allocation outcomes. C: Relationship between risk-adjusted returns (Sharpe ratio) and realized drawdowns. D: Relationship between the two portfolios highlighting portfolios that achieve strong Sharpe ratios while keeping maximum losses relatively contained.*

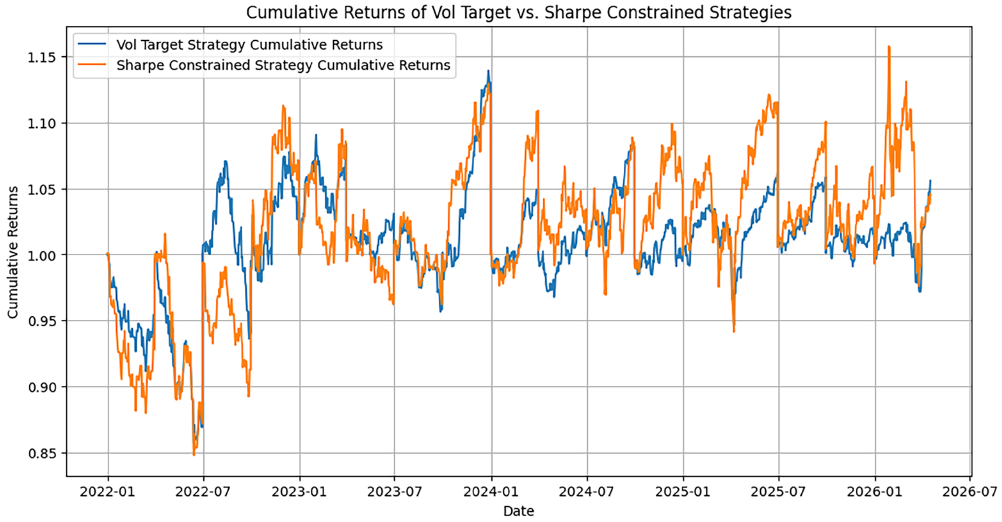

Figure 8. *Comparison of cumulative returns of S1 and S2 over the period 2022 to 2026, illustrating their relative performance over time. The x-axis represents the time horizon, while the y-axis shows cumulative returns.*

S1 methodology. Sharpe ratios fluctuated across both strategies, highlighting that risk-adjusted performance is sensitive to market conditions and that volatility control is more critical than consistent Sharpe stability. Volatility and return did not exhibit a consistent linear relationship in this sample, suggesting that higher returns were not systematically associated with higher volatility. MaxDD and Sharpe ratios exhibited a general inverse relationship: assets with smaller drawdowns tended to have higher Sharpe ratios, indicating that optimal portfolios balance upside potential with controlled downside exposure.

The results indicate that dynamic risk management and exposure control are crucial for long-term portfolio performance. High-volatility assets, such as BTC-USD, can enhance returns but require careful allocation to mitigate tail risk. Volatility-targeted strategies provide stability and consistent risk-adjusted performance, while Sharpe-constrained strategies capture upside in strong markets but are more susceptible to losses during downturns. On average, S2 achieved higher annualized returns and Sharpe ratios during the recovery period (2023-2025), while S1 maintained consistently lower volatility and smaller maximum drawdowns across all periods (Figure 8). These findings support the hypothesis that high-volatility assets do not inherently guarantee superior risk-adjusted returns and highlight the importance of incorporating volatility forecasting, tail-risk measures, and dynamic rebalancing in portfolio optimization.

CONCLUSION

This study demonstrates that dynamic, multi-asset portfolio optimization can meaningfully improve the risk–return tradeoff compared to static approaches, although outcomes are highly dependent on the chosen strategy. The hypothesis that high-volatility assets outperform lower-volatility portfolios was only partially supported. Results indicate that high-volatility asset classes can deliver superior returns only when their exposure is actively managed; they do not consistently provide higher risk-adjusted returns in the absence of effective risk control.

The volatility-targeted portfolio (S1) produced more stable performance, lower volatility, and smaller drawdowns, emphasizing the benefits of S1 approaches in managing downside exposure. Conversely, the Sharpe-constrained portfolio (S2) achieved higher returns and stronger risk-adjusted performance during rising markets, but exhibited greater vulnerability to substantial losses during market downturns. These findings suggest that dynamic risk management and active rebalancing are the dominant drivers of long-term portfolio performance, rather than volatility alone. Incorporating advanced risk metrics, such as GARCH-based volatility forecasting, VaR, CVaR, and Sharpe/Treynor ratios, allows for more informed allocation decisions and enhances portfolio resilience across varying market conditions.

This study has several limitations. The analysis spans a relatively short period (2021–2025), which may not fully reflect long-term market dynamics. This interval also includes atypical conditions such as post-pandemic recovery and elevated inflation, which may limit the broader generalizability of the findings to more stable, long-term market environments. Additionally, the selected asset universe is limited, and results may differ with a broader or alternative set of assets. Future research could extend the analysis to longer historical periods and incorporate a more comprehensive range of global asset classes to validate and generalize these findings.

This study contributes to the existing literature by providing an empirical comparison of volatility-targeting and Sharpe-constrained portfolio strategies within a unified GARCH-based framework. By integrating time-varying volatility dynamics into the optimization process, the analysis captures more realistic market behavior and allows for a consistent evaluation of both approaches under identical modeling assumptions. Beyond simply comparing performance outcomes, the study emphasizes the inherent trade-offs between risk stability and return maximization in dynamic portfolio construction. Volatility-targeting strategies are shown to prioritize downside protection and smoother return profiles, often at the expense of foregone upside potential. In contrast, Sharpe-constrained approaches tend to tilt toward higher risk-adjusted returns, but may exhibit greater sensitivity to market regimes and periods of heightened volatility. By situating both strategies within the same econometric structure and evaluating them across different market phases, the study offers a more controlled and comparable assessment than much of the existing literature. This unified approach helps clarify how different optimization objectives influence portfolio behavior over time, particularly under changing volatility conditions. Overall, the findings provide practical insights for portfolio managers in balancing competing objectives of risk control and return enhancement, and contribute to a deeper understanding of how constraint design shapes dynamic ass*et al*location outcomes in volatile markets.

CONFLICT OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this article.

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