Original Research Article

Cross-National Differences in the Pedagogy of Fundamental Algebra: A Comparative Review of Textbooks from Three Countries

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ABSTRACT

This study examines differences in the textbook teaching of basic algebra in three different countries. Primary sources included nationally published curricula and popular algebra textbooks from the United States, United Kingdom, and India. To facilitate comparison, the chapters reviewing equations with parenthetical expressions were selected for analysis. Common to all textbook examples were the age of the intended audience and the introduction of three principles: the distributive property, simplification of algebraic expression by combining like terms, and reduction by the greatest common factor. The curricula differed, however, in the order in which the topics were presented, and this corresponded to differences in the approach to problem sets, namely in the order of operations recommended. Further study is needed to determine if these observed variations in textbook approach become more pronounced in advanced algebraic curricula, and whether this leads to any appreciable inter-country differences in problem-solving methodology or mathematical reasoning between student populations.

Keywords: Algebra Curriculum; Mathematics Pedagogy; Distributive Property

INTRODUCTION

The subject of human knowledge acquisition has received renewed focus in recent times, particularly as it relates to the learning of math, science, and technology (1). In order to approach this issue from a global educational perspective, we must first know whether humans around the world are taught math in the same way or in different ways. With respect to the pedagogy of basic algebra, there are two sequential questions: is there inter-country variation in the mathematical principles taught or the manner and sequence in which they are introduced? If there are differences, do they impact how problem-solving is approached? To answer these questions, we reviewed primary textbooks in three disparate global regions, focusing on a single, discrete topic—the introduction of parenthetical expressions in basic algebraic problem-solving.

METHODS AND MATERIALS

One country from each of three continents, Europe, Asia and North America, was identified. The United Kingdom and India were selected from their respective continents based on the availability of a national standardized curriculum in English. The government of India, through its National Council of Educational Research and Training, provides free global and public access to all textbooks associated with the national curriculum in multiple languages, including English. *Ganita Prakash Textbook of Mathematics for Grade 7* was accessed online for this study (2). Similarly, the

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UK National Curriculum was also accessed online (3). A variety of textbooks that cover the National Curriculum are available to schools in the United Kingdom, with the Oxford and White Rose series being the most widely used (4-6). English language textbooks for both USA and Canada were easily accessed, however the United States was selected for analysis due to the availability of texts used widely across geographic regions, whereas in Canada, textbook preferences differ sharply by province. The most popular of the American textbooks, *Glencoe Pre-Algebra* (7) and *Prentice Hall Mathematics Pre-Algebr*a (8) were utilized in this study.

Primary textbooks for grade school children were reviewed in India, the United States, and the United Kingdom. The introductory chapter on algebraic equations was identified in each curriculum, alongside the age group for which the chapter was written. Age of readership determination was necessary to address the idea that any variations in teachings might be attributable to differences in the developmental stage of the intended audience. Furthermore, to facilitate comparison, this study identified a discrete topic found in the first half of each of the three curricula on fundamental algebra: the introduction of parenthetical expressions in basic algebraic problem-solving. To illustrate the three different approaches, three equations were drafted, each resembling a basic problem-type reviewed in the chapters related to equations with parentheticals. The methods of each of the textbooks were employed individually to arrive at the solution to these problems, while demonstrating differences in approach.

RESULTS

Differences in the Age Demographics of Students Learning Fundamental Algebra

Though the central aim of this research was to evaluate differences in approach in the teaching of basic algebra, in order to place the textbooks in the proper context of their use, the age of the students using these textbooks was determined. In the United States, introduction to algebra, called "pre-algebra," including the topic of parenthetical expressions, was found in textbooks used during the sixth and seventh grades, corresponding to ages 11-13. In India, this same topic is introduced in the seventh class, corresponding to the same age range of 11-13. In the United Kingdom, basic algebra is learned in Key Stage 3, years 7 and 8, also corresponding to ages 11-13.

Differences in the Ordering of Lesson Topics

To solve equations with simple parenthetical equations, one or more of three operations must be employed prior to isolating the variable: utilization of the distributive property, simplification by combining like terms, and reduction by the greatest common factor. All three curricula introduce these lessons but not in the same order (Table 1).

In the UK national curriculum, these topics all fall under the same learning objective, namely that "pupils should be taught to simplify and manipulate algebraic expressions to maintain equivalence" (3). In the White Rose textbooks (4, 5), reduction by the greatest common factor is taught first in the chapter "Using the Equals Sign." Next comes simplification in the chapter "Like and Unlike Terms." Finally, the distributive property is explained in the chapter, "Brackets and Equations."

By contrast, in the American curriculum, the distributive property is taught early in both the Glencoe and Prentice Hall textbooks, followed by simplification of algebraic expression. Reduction by the greatest common factor is taught last, in the context of solving equations and isolating the variable (7, 8).

Lastly, in the Indian curriculum, attention is first given to achieving "the simplified form" of every algebraic expression prior to solving. Later, in this same chapter, use of the distributive property with respect to variables is taught. Reduction by the "highest common factor"

Table 1. *Order in which Mathematical Principles are Taught, By Country*

	USA	UK	India
First Lesson	Distributive Property	Reduction by Greatest Common Factor	Simplification/Combining Like Terms
Second Lesson	Simplification/Combining Like Terms	Simplification/Combining Like Terms	Distributive Property
Third Lesson	Reduction by Greatest Common Factor	Distributive Property	Reduction by Greatest Common Factor

is a much later lesson in the text, in a chapter labeled "Finding the Unknown," and, like in the American curriculum, is discussed in the context of isolating the variable and solving the equation (2).

Differences in the Approach to Problem Solving

While all three of the operations discussed were employed in problem sets, the solutions and explanatory notes demonstrated a difference in priority of operations, corresponding, in fact, to the ordering of lessons in the text (2-8). This finding is best illustrated by a review of equations which require at least two of these operations to solve. Consider an equation with a parenthetical expression and a non-reduced greatest common factor: 2(x+3) = 12.

There are two ways to approach this problem. Method A, used by the Americans and Indians, uses the distributive property first, then reduction by the greatest common factor. Method B, used by the British, reverses this order (Table 2). Next, consider an equation with a parenthetical expression and non-consolidated like terms: 3 (x+2) + 4 (x +2) + 3 = 31

In this case, Method A, used by the Americans, involves distributing first, then simplifying by consolidating like terms. Method B, used by the British and the Indians, reverses this order (Table 3). Finally, consider the equation with a parenthetical expression, a non-reduced greatest common factor, and non-consolidated like terms: 3 (x+2) + 6(x+2) + 9 = 45.

There are numerous ways to approach this problem, however to focus on the three employed by the three countries (Table 4), Method A, the American approach, involves distributing first, then simplifying. Method B, used in the British examples, first reduce by the greatest common factor, then simplify by consolidating like terms. In this case this sequence is repeated, given the priority of reduction and simplification, obviating the need for the distributive property. Finally, Method C, the Indian approach is to simplify first, then use the distributive property.

Table 5 summarizes the priority of algebraic operations by country, illustrated by the sample problems.

Note that in the event that the equation has no like terms to combine and simplification is not required, the distributive property becomes the first step and reduction by the greatest factor the second for both the USA and India methods (Table 2, Table 5). Likewise, in the event that there are no common factors to reduce, simplification becomes the first step for the UK method, and distribution the second, making this approach identical to the one used in India (Table 3, Table 5). The

Table 2. *Approach to the Equation with a Parenthetical and a Greatest Common Factor*

METHOD A (USA, India)	METHOD B (UK)
2 (x+3) = 12	2 (x+3) = 12
2x + 6 = 12	x + 3 = 6
x + 3 = 6	x = 3
x = 3	

Table 3. *Approach to the Equation with a Parenthetical and Like Terms*

METHOD A (USA)	METHOD B (UK, India)
3 (x+2) + 4 (x +2) + 3 = 31	3 (x+2) + 4 (x +2) + 3 = 31
3x +6 + 4x + 8 + 3= 31	7 (x + 2) + 3 = 31
7x + 17 = 31	7x + 14 + 3 = 31
7x = 14	7x + 17 = 31
x = 2	7x = 14
	x = 2

Table 4. *Approach to the Equation with a Parenthetical, Like Terms, and a Greatest Common Factor*

Method A (USA)	Method B (UK)	Method C (India)
3 (x+2) + 6(x+2) + 9 = 45	3 (x+2) + 6(x+2) + 9 = 45	3 (x+2) + 6(x+2) + 9 = 45
3x + 6 + 6x + 12 + 9 = 45	(x +2) + 2 (x+2) + 3 = 15	9 (x+2) + 9 = 45
9x + 27 = 45	3 (x +2) + 3 = 15	9x + 18 + 9 = 45
9x = 18	x + 2 + 1 = 5	9x + 27 = 45
x = 2	x+3 = 5	9x = 18
	x = 2	x = 2

Table 5. *Priority of Algebraic Operations by Country*

	USA	UK	India
First Operation	Use of the Distributive Property	Reduction by the Greatest Common Factor	Simplification by Combining Like Terms
Second Operation	Simplification by Combining Like Terms	Simplification by Combining Like Terms	Use of the Distributive Property
Third Operation	Reduction by the Greatest Common Factor	Use of the Distributed Property	Reduction by the Greatest Common Factor

last equation, involving both like terms and greatest common factors, highlights the three distinct priorities of operations by country, corresponding also to the order in which these mathematical principles were introduced to students (Table 1, Table 4, Table 5).

DISCUSSION

There are limits to the generalizability of educational standards to all students within any single country. For example, while the Indian text used was the primary textbook across a very standardized public education system, only a little over half of all Indian children are enrolled in the public system (9). Private schools, including those that advertise a "Western education," do not uniformly use the same standardized text, making it difficult to make any assertions about the experience of children in private schools. In the United Kingdom, by contrast, over 90% of students are enrolled in the "state" system which has a standardized National Curriculum (3), though textbooks are not uniform. Similarly, in the United States, schools, both private and public, choose from many different textbook offerings, with decentralized decisions made at the individual school and community level (10). Although the textbooks analyzed in this study had high market penetration and the teachings appeared standard across the most popular American textbooks, no one textbook was used in the majority of classrooms, limiting generalizability of the American student experience.

There was consistency among all three countries' curricula in the use of a stepwise approach to problem solving. All textbooks reviewed taught three lessons central to this problem-solving: the distributive property, simplification by combining like terms, and reduction by the greatest common factor. The order in which these topics were presented differed, however, and, more importantly, these differences persisted in each country's unique approach to determining which steps took priority when solving equations.

One additional surprising finding was that the specific concept of parenthetical expression in basic algebra was introduced to students at the same age across the curricula of all three countries. This contrasts with the widely held belief, among some Westerners, that non-Western math education introduces algebra at an earlier age (11, 12). Of note, age differences might have been observed if a different non-Western country had been chosen. Notably Russian Davydov and Chinese mastery schools introduce algebraic concepts early in childhood, during the teaching of elementary arithmetic (12, 13).

It is important to note that although one country on each of three continents was selected for analysis, the educational principles observed, and the educational experience itself, are not generalizable throughout the continent. For example, the Canadian educational system routinely outperforms its American neighbors (10). Additionally, although some have noted strong similarities among the educational systems of the Western European countries—a result, perhaps, of international cooperation—Eastern European countries are believed to have an approach to math education that is distinct from the Western curricula (14). Likewise, the Indian experience is not generalizable to Asia. In fact, several Asian countries are credited with having very distinct schools of mathematical thought including the Chinese Shanghai Math Method and what is commonly referred to as "Singapore Math." In addition, India's long and recent colonization history suggests that India's curriculum may have fewer differences from Western teachings when compared with its Asian neighbors. Prominent distinction from Western pedagogy is observed, for example, in never-colonized Japan (15).

This comparative study evaluated only differences in the texts used in classrooms and therefore is limited in its ability to characterize the student experience. Other factors, such as teacher preparedness and classroom implementation, are reported to have a significant impact on variations in classroom learning between countries and are not evaluated here (14-16). Though it has been argued that even subtle variations in pedagogy may lead to significant divergence in mathematical reasoning between populations (17), rigorous study of student outcomes would be required to make any such assertion.

CONCLUSION

While there was no difference in the ages of students being taught basic algebra, there were variations between countries in the order in which the relevant mathematical principles were presented. Moreover, these variations corresponded to differences in approach to solving basic algebraic equations, namely in the order of recommended operations. This study provides support for a link between a subtle pedagogical difference (i.e. order in which mathematical equation-solving principles are taught) and differences in the way problems are approached in each country's textbooks. Additional study is needed to determine if these differences are seen in the equation-solving practices of the actual students using the textbooks, and whether these inter-country variations become more or less pronounced in advanced algebraic curricula. Finally, further testing of students would be required to determine whether these pedagogical differences impact mathematical reasoning, and if so, whether the development of human mathematical reasoning would benefit from determining an optimal strategy and standardizing it globally. Alternatively, perhaps maintaining or accentuating these differences would be best, with individuals benefiting from learning from multiple different schools of thought, and, likewise, teams of mathematicians benefiting from diverse international representation. It remains to be determined, in an era of increasing globalization, whether pedagogical differences between countries are a weakness to be improved upon, or a strength to be embraced.

CONFLICT OF INTEREST

The author declares that there are no conflicts of interest related to this work.

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