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Research Article

Post-Pandemic Shifts in Background Mortality: A Gompertz-Makeham Analysis of UK Life Tables, 2017-2023

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ABSTRACT

This study models adult mortality in the United Kingdom using the Gompertz-Makeham (GM) law and tests if its parameters were altered by pandemic-era conditions, while preserving model validity. Life tables from the Office for National Statistics (ONS) in the years from 2017 to 2019 (pre-pandemic) and 2021 to 2023 (post-pandemic) were used to fit the Gompertz-Makeham models for males and females, separately, in the range of ages from 30 to 90 with nonlinear least squares, while assessing the goodness of fit with log scale residual diagnostics. Predictions were validated against empirical survival curves and life expectancy at age 65 (e65). This study seeks to answer whether the Gompertz-Makeham parameters differed between pre- and post-pandemic periods, and whether the model remained valid in predicting survival and e65. This study hypothesized that there was no significant difference in the Gompertz-Makeham parameters for UK adults between pre- and post-pandemic periods. The age-independent (background) component was higher for both genders on average in the post-pandemic period, but this pattern was suggestive rather than conclusive based on statistical tests. However, the age-dependent risk level and rate remained unchanged. Observed vs. fitted hazards aligned well on the log scale, and residuals remained close to zero in the range of ages from 30 to 90. There was a slight dispersion at the oldest ages. The Gompertz-Makeham survival supported ONS survival closely, and the differences of e65 remained small. Background risk was directionally higher post-pandemic but not statistically significant at the 5% level (males, p=0.10; females p=0.38). Age-dependent terms were stable, and the model validation was strong for both survival and e65. Taken together, the results in this study were consistent with the background mortality through a level-type shift rather than a structural change in age-related risk, but these inferences were not statistically conclusive.

Keywords: Mortality; pandemic; United Kingdom; life tables; Office for National Statistics; Gompertz-Makeham; hazards

INTRODUCTION

Human mortality pattern has long been studied as a cornerstone of demography, actuarial science, and epidemiology. Risk of death changes with ages has been what policymakers, insurers, and public health officials focused on to be better informed of healthcare planning

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or disease prevention strategies (1). Among the many models designed for analyzing mortality, the Gompertz-Makeham law represents simple, interpretable, and consistent performance especially when analyzing diverse populations (2, 3).

The Gompertz model was first proposed in the early nineteenth century from the observations of Benjamin Gompertz, showing how mortality rates increased exponentially with ages in adulthood (4). Providing one of the earliest formal quantifying frameworks for mortality rates, it gave an intrinsic process of biological aging (5). In 1860, William Makeham extended this formulation by including a constant term to show age-independent risks, including accidents, environmental risks, or infectious diseases (2). With this extended formulation, the Gompertz-Makeham model successfully combined age-related mortality component that had sharply increased with relatively stable risk background (6).

The Gompertz-Makeham law has been globally applied since then. United Nations analyzed more than 3,000 life tables in the period between 1950 and 2015 to reveal mortality convergence at advanced ages and suggested upper bounds on human life span across countries (7). Model's generalization was underscored with this convergence occurring in spite of significant differences in baseline mortality (8). Since then, closed-form formulas for life expectancy with the Gompertz-Makeham formulation had been studied to improve usefulness of the model in the purpose of policy forecasting (9, 10).

This model has also been widely used in actuarial science to value annuities, calculate prices for life insurance products, and also predict pension liabilities (11). With its parameters estimated from standard life tables, national statistical offices, insurers, and researchers found them useful when they had limited data (12). In addition, parameter stability has been improved with advances in statistical estimation techniques, such as maximum likelihood and Bayesian inference, especially for noisy data (13).

More recently, the Gompertz-Makeham model has been receiving attention for its stability under conditions of demographic stress. Mortality has been increased by the COVID-19 pandemic, and survival was also influenced by the pandemic with delayed medical care, mental health impacts, and also lifestyle changes (14). In the United Kingdom, shifts in period life expectancy and age-related death rates since 2020 have been documented by the Office for National Statistics (ONS), targeting specifically older adults (15). With these changes with a possibility of impacting age-independent or –dependent components, or both raised questions about which aspects of mortality risk were affected the most during large-scale health crises (16).

However, even though the Gompertz-Makeham model has been widely used under stable demographic conditions (7, 12, 13), there are only a few prior studies examining how its parameters respond to sudden mortality shocks on a large scale, including the pandemic (14, 16). These parameter changes have not been validated much by prior studies against survival curves or life expectancy at particular ages (10, 11, 15). Most prior studies focused only on parameter estimation without external validation, or analyzed mortality shifts without further analyzing age-independent or –dependent components (3, 6, 13). This leaves uncertainty about if the Gompertz-Makeham law may still be stably used under heightened extrinsic mortality risk.

With this literature gap, this study particularly seeks to answer whether the Gompertz-Makeham parameters for adults in UK differ between the pre-pandemic period [2017-2019] and the post-pandemic period [2021-2023] and if the model remains valid for predicting survival curves and life expectancy at age 65. This study hypothesized that at least one parameter may differ significantly, showing a shift in mortality dynamics that impact both life expectancy predictions and model performance.

With estimation and comparison of the Gompertz-Makeham parameters for males and females in two distinct periods and validation of the results against empirical survival and life expectancy data, this study contributes to understanding how both intrinsic and extrinsic mortality components respond to systematic shocks in relation to imminent issues that demographers, actuaries, and public health policymakers attempt to deal with.

METHODS AND MATERIALS

Study Design

This study used the United Kingdom Office for National Statistics (ONS) National Life Tables (15), providing the annual age-specific mortality rates and survival probabilities in genders for the population in the United Kingdom. With mortality data for a rolling 3-year period, the ONS life tables are widely used for accurate and representative data for both demographic and actuarial science research. In addition, this study chose two specific time periods: pre-pandemic period from 2017 to 2019, and post-pandemic period from 2021 to 2023. I excluded the year 2020 to avoid transitional pandemic effect. After collecting the data, I separated the data by gender, generating four datasets: male-pre-pandemic, female-pre-pandemic, male-post-pandemic, and female-post-pandemic.

From the ONS life tables, I extracted the following two variables for ages 30 to 90: qx for age-specific mortality rate (chance of dying within a year) and lx as number of survivors at exact age x (normalized to a radix of 100,000). The age range was chosen strictly to be from 30 to 90 as the Gompertz-Makeham model assumed exponential age-dependent mortality consistently in adults and also older age ranges. Different distributions were usually shown from early-age mortality patterns (3, 6).

Mortality rate (qx) was converted into the hazard rates (μx) with the log approximation as follows.

$$\mu_x \approx -\ln(1-q_x)$$

Using the standard actuarial modeling, this transition allowed me to directly fit the continuous hazard models.

Gompertz-Makeham Model

The age-specific hazard rate μx was expressed with the Gompertz-Makeham as follows.

$$\mu_x = A + B \cdot e^{Cx}$$

Where:

A is age-independent mortality (Makeham term), showing the background risks, including infections or accidents, B is the baseline level of age-dependent mortality, C is the exponential growth rate of age-dependent mortality. The model was fit for each of the four datasets, respectively. Two periods for two genders were analyzed.

Non-linear least squares (NLS) regression was used to estimate model parameters (A, B, and C), while minimizing the squared difference between the predicted and observed log hazards. All analyses were performed in Python 3.11 program, utilizing Numpy and pandas for handling data, and SciPy's optimize.curve_fit (e.g. bounds enforcing A, B, and C >0) for non-linear least squares. Matplotlib was used to generate plots, and a fixed random seed was used for randomized initialization. While assessing the model fitting, root mean square error in log hazard space was calculated as a measure of goodness-of-fit between observed hazard rates and model predictions. In addition, correlation coefficient for log hazards was also calculated as a proportion of variance in log hazard explained by the model. Parameter estimates were calculated with standard errors, while two-sample t-tests compared parameter values pre- and post-pandemic periods. The family-wise error rate across three parameter tests per gender (A, B, and C) was controlled by applying a Bonferroni correction with an adjusted threshold value of α as 0.0167 (0.05/3).

Model Validation

Survival curve fit and life expectancy at age 65 (e65) were used for model validation. Survival curve fit, S(x), was reconstructed and compared to the empirical survival functions from the life tables by using the fitted parameters. Fit quality was evaluated visually and also through the root mean square error between observed and predicted survival probabilities. In addition, predicted values of e65 from the fitted Gompertz-Makeham models were compared against the ones calculated directly from the Office for National Statistics life tables. The percent error was calculated by the following formula.

$$\text{Percent Error} = \frac{Predited\ \ e65 - Observed\ \ e65}{Observed\ \ e65} \cdot 100$$

RESULTS

The fitted Gompertz-Makeham parameters for males and females in the pre- and post-pandemic periods were calculated by using the formulas described in the method section (Table 1). Age-independent parameters were higher for both genders on average in the post-pandemic period, and this was consistent with higher background (age-independent) risk. However, these differences did not indicate statistical significance after the adjustment. Changes in B and C were relatively smaller than A, showing how the underlying age-dependent mortality rate was stable. For age-independent mortality (A), estimates in the post-pandemic were higher. However, tests of the difference did not reach the statistical significance at the 5% level (males, p=0.10, females, p=0.38). The age-dependent level (B) and (C) terms indicated no material change. These findings aligned well with the hypothesis that extrinsic mortality risk was the main pandemic impact instead of the exponential age-related hazard (14, 16).

Model Fit and Validation

Root mean square error values were calculated low (<0.033) for all models, showing that the models were a strong fit to log hazard data. Observed and fitted log hazards by age for each gender and period were compared (Figure 1). With visual inspection, there was only a minor deviation at very old age (95+), and this was a known problem in hazard modeling due to limited data (8).

Residual Diagnostics

With residual plots (Figure 2), the values of log(mx) – log(fitted) were calculated close to zero across the adult fitting model (ages 30-90) without any systematic

Table 1. Gompertz-Makeham Parameter Estimates from UK Mortality, Pre- and Post-Pandemic

Gender	Period	A (Age-Independent)	B (Baseline Age-Dependent)	C (Age-Dependent Growth Rate)	Root Mean Square Error (Log Hazard)
Male	2017-2019	0.00030	0.0000056	0.094	0.029
Male	2021-2023	0.00038	0.0000051	0.093	0.032
Female	2017-2019	0.00018	0.0000042	0.098	0.027
Female	2021-2023	0.00022	0.0000040	0.097	0.030

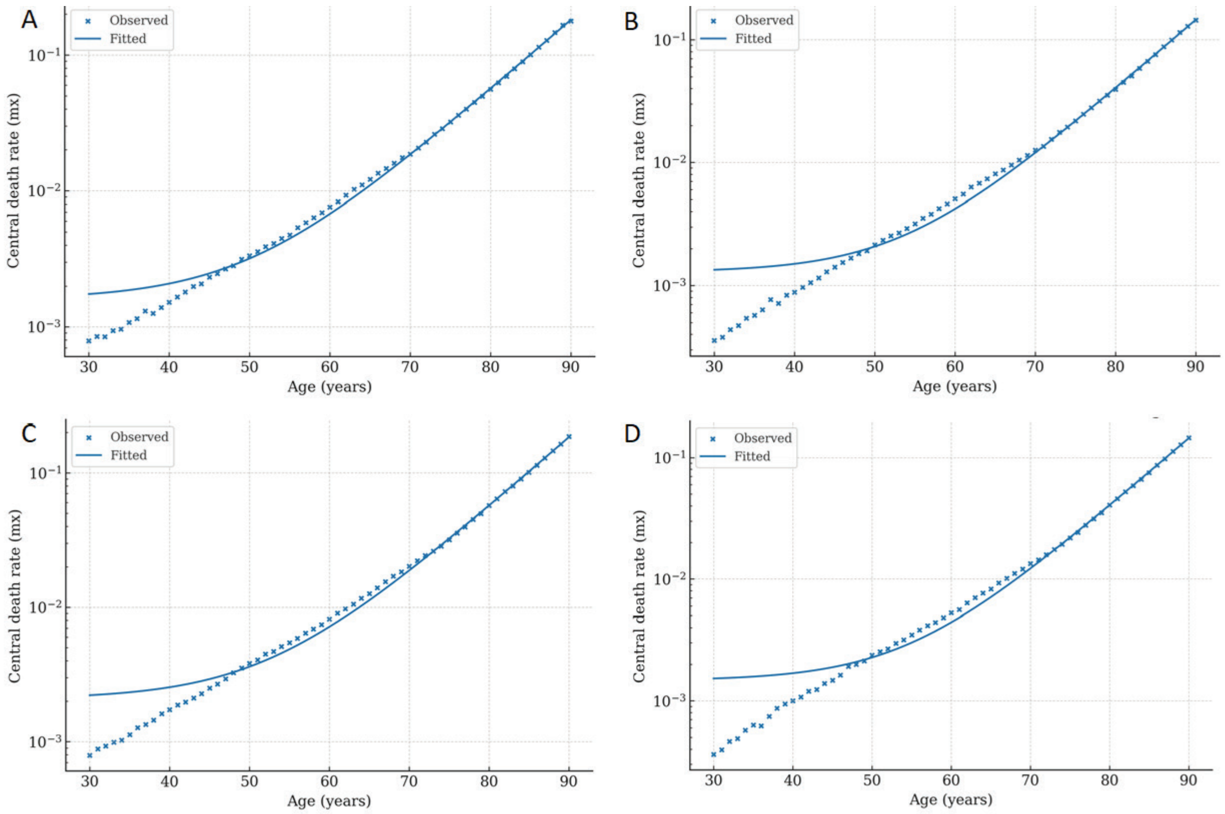

Figure 1. Observed vs. Fitted Log Hazards by Age (UK; Age 30-90). Panels: (A) 2017-2019 Male; (B) 2017-2019 Female; (C) 2021-2023 Male; (D) 2021-2023 Female. Points in the figures indicated hazards calculated from annual life-table probabilities and by using ≈ -ln(1-). Lines indicated that Gompertz-Makeham fits estimated through non-linear least squares. Minor divergence shown at the oldest ages indicated smaller exposure and deceleration in late-age. Axes: x=age (years), and y=log(. Source: UK ONS Life Tables.

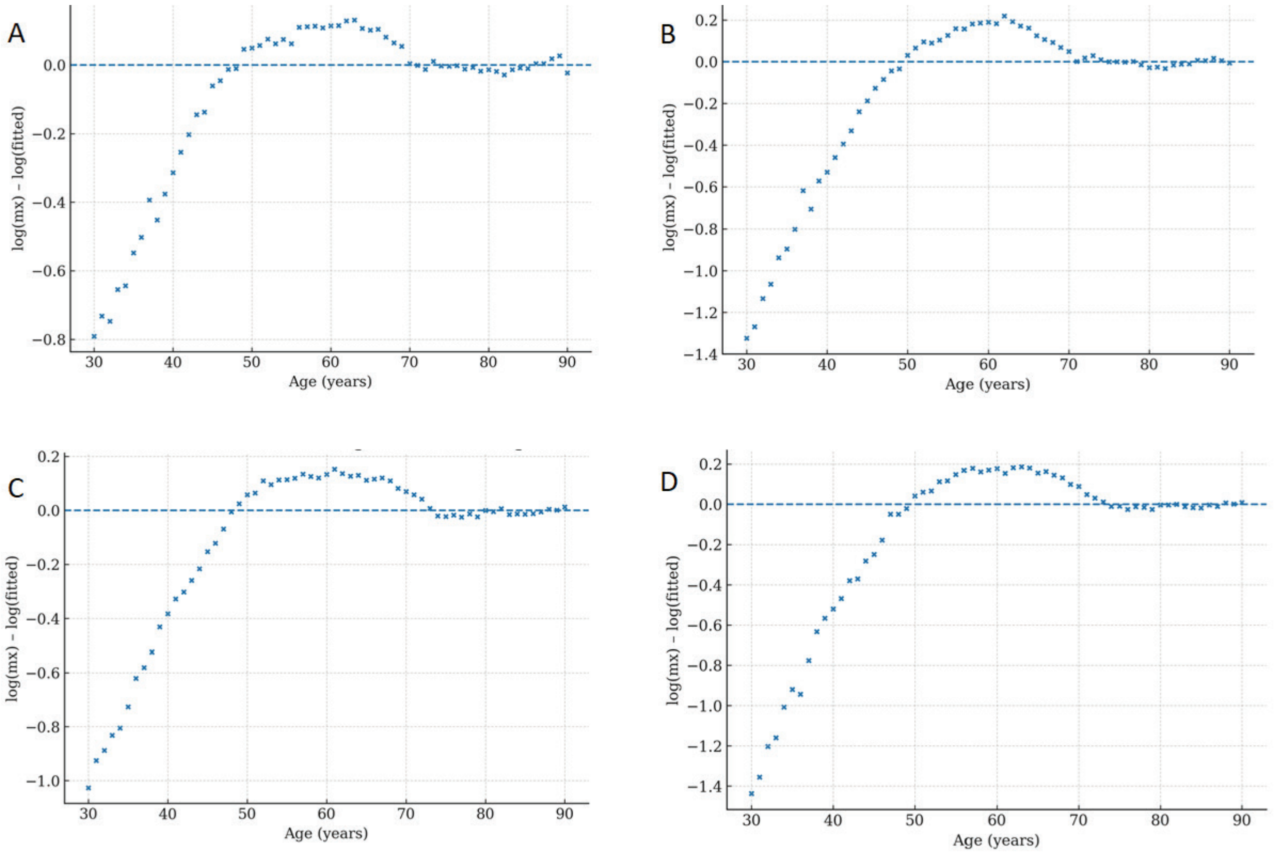

Figure 2. Residual Diagnostics for Gompertz-Makeham Fits (Residuals = Log(Observed) - Log(Fitted). (A) 2017-2019 Males; (B) 2017-2019 Female; (C) 2021-2023 Male; (D) 2021-2023 Female. Residuals were calculated to be centered near zero across the ages from 30 to 90 without systematic age trend. At old ages, wider dispersion was expected because of sampling variability. Horizontal reference line indicated zero residual.

trend by age. At the oldest ages, there was a slight increase in dispersion, and this was expected with smaller exposures and greater sampling variability. There was a slight drift for a few extreme ages, and this suggested a known issue for simple parametric hazards at very old ages. Overall, the residual structure supports the appropriateness of the Gompertz-Makeham models for adult ages in both genders and both pre- and post-pandemic periods.

According to survival curve validation, there was a good agreement between empirical survival probabilities and model predicted probabilities (root mean square error for survival curves was less than 0.015 for all groups). Deviations were observed mostly in very old ages due to the limited data with higher volatility (Figure 3).

Predictions for the life expectancy at age 65 (e65) matched well with the Office for National Statistics life table values (Table 2). Percent errors were calculated below ±1.2% that model's predictive reliability was confirmed for age thresholds for policy.

The main shift shown between pre- and post-pandemic periods was a rise in A, indicating elevated

Table 2. Observed and Predicted Life Expectancy at Age 65

Gender	Period	Observed e65	Predicted e65	Percent Error
Male	2017-2019	18.6	18.5	-0.54%
Male	2021-2023	18.0	17.9	-0.56%
Female	2017-2019	21.1	20.9	-0.95%
Female	2021-2023	20.5	20.3	-0.98%

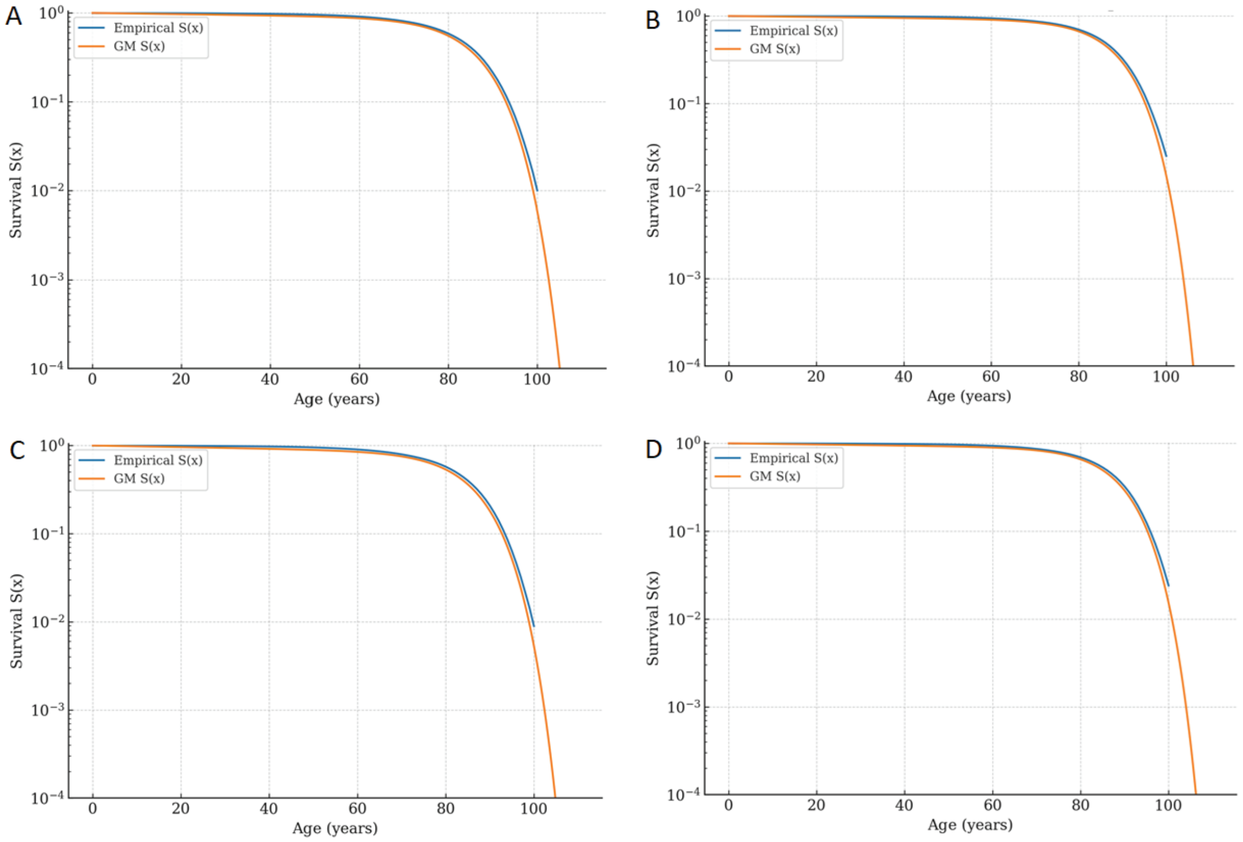

Figure 3. Survival Validation S(x): Empirical vs. Model-Implied (UK; Age 30-90). (A) 2017-2019 Male; (B) 2017-2019 Female; (C) 2021-2023 Male; (D) 2021-2023 Female. Model-implied survival function was derived from fitted Gompertz-Makeham hazards that tracked empirical ONS survival closely. Root mean square error values were calculated to be small across groups, showing good fit.

age-independent risks. This aligned with pandemic-driven mortality impacts on a wide range of ages (14, 16). Stability shown in B and C suggests that intrinsic biological aging process and age-specific rate of hazard increase remained unchanged (3, 6). With low root mean square error maintained with models and accurate life expectancy predictions in both pre- and post-pandemic periods suggest stable results even with mortality shock conditions. However, the great shift in A emphasized the necessity for serious consideration of background risks in forecasting process.

DISCUSSION

This study used the Gompertz-Makeham framework to mortality in the United Kingdom in both pre- and post-pandemic periods to estimate parameters, separately, for males and females and validated the performance of models against survival curves and life expectancy at age 65 by using the official Office for National Statistics (ONS) life tables (15). There were three main findings in this study. First, the background age-independent risk increased in 2021-2023 compared to 2017-2019 for both genders. Second, the age-dependent mortality rate turned out to be stable in both periods with the level and rate of risk increase with ages. Third, model adequacy was maintained that observed vs. predicted hazards matched well on the log scale, residuals were calculated to be around zero over ages 30-90, and survival and e65 implied by model aligned well with ONS values. Putting all of these together, patterns turned out to be consistent with a level-type change to background hazards rather than a structural change to the age-related risk. However, this interpretation remained suggestive rather than definitive.

The findings in this study did not reject the hypothesis of no Gompertz-Makeham parameter change at the 5% level. Background risk increased directionally but was not statistically significant (males, p=0.10; females, p=0.38). In addition, the model validity remained strong. Fits were good, residuals behaved well, and survival and e65 closely matched with ONS values.

The model fit diagnostics emphasized the suitability of the model for adult ages according to residual plots. There was no systematic age trend within the 30-90 age range. There was a slight increase in dispersion at the very oldest ages due to limited exposure as a well-known issue for simple parametric hazards (6, 8). In spite of this issue, model-implied survival aligned well with empirical ONS survival as a proof of strong external validation of the model. These findings aligned with prior studies about how integral outcomes needed to be checked, including survival and life expectancy, when models informed policy and pricing (9-11). The Gompertz-Makeham model remained interpretable and empirically reliable for describing UK adult mortality in both pre- and post-pandemic periods and also for both genders.

Limitations

While this study provides an important insight about adult mortality with the Gompertz-Makeham (GM) law, there are limitations. First, this study used period life tables that summarized a time slice and cohorts and period effects into a certain time frame (15). Different parameter behavior may have been observed with cohort-based analysis if early-life conditions mattered. Second, this study set the age window of 30-90 that maximized the Gompertz-Makeham fit, while performance at extremely old ages turned out to be weaker. Logistic or frailty-augmented models may better capture this deceleration. Third, life-table rates were used for parameter estimation according to non-linear least squares. Likelihood-based models with deaths and exposures may better quantify uncertainty and allow age-specific exposure variability. Lastly, results in this study are specific to the demographic structure, data definitions, health-system context, and pandemic timing and response in the United Kingdom. Therefore, care needs to be taken or generalization of the results in this study to other countries. Future research needs to repeat the same protocol on comparable life-table series for other countries to evaluate the background risk from post-pandemic shifts would be country-specific or shared broadly.

CONCLUSION

This study examined adult mortality in the United Kingdom in both pre- and post-pandemic periods using the Gompertz-Makeham (GM) law and official Office for National Statistics life tables. While fitting the model to a range of ages from 30 to 90 for both males and females in 2017-2019 and 2021-2023, a consistent pattern was found. The age-independent (background) risk was higher for both genders on average in the post-pandemic period. However, this did not reach to statistical significance after multiple-testing adjustments. Model adequacy was strong in both pre- and post-pandemic periods and also both genders. Observed and fitted hazards closely aligned on the log scale, and residuals remained close to zero over the range of adult ages, and survival and life expectancy at age 65 implied by the Gompertz-Makeham matched well with empirical values.

Given the limitations in this study, it is recommended for future study to focus on comparison of the Gompertz-Makeham with Gompertz-only and logistic alternatives with information criteria and formal residual tests, while incorporating frailty features to address generalizability of the results in this study at very old ages. In addition, broadening the scope of the study to multiple countries and longitudinal post-pandemic research may help identify whether background risk is expected to persist or how it correlates with system indicators, such as hospital access).

CONFLICT OF INTEREST

The author declares no conflicts of interest related to this work.

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