

The Measurement Problem and the Sebens-Carroll Derivation of the Born Rule

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ABSTRACT

The measurement problem remains one of the most profound puzzles in the foundation of quantum mechanics. It contains two distinct aspects: the collapse of the wavefunction and the origin of probabilities. While Everettian Quantum Mechanics (EQM) and the Many Worlds Interpretation (MWI) work around the collapse aspect, by positing that the universe branches through pure unitary evolution, it leaves unresolved the question of why measurement outcomes occur with frequencies given by the Born Rule. This paper explores a recent derivation of the Born Rule by Carroll and Sebens based on principles of Self-Locating Uncertainty (SLU) and the Epistemic Separability Principle (ESP). By examining this framework in detail, beginning with an equal amplitude case and extending to an unequal amplitude case, we show how these principles account for the probabilities given by the Born Rule, hence showing that SLU and the ESP, within the scope of EQM, offer a solution to the measurement problem. We further show the significance and implications of deriving the Born Rule and highlight the relevance of the measurement problem to more recent developments like quantum gravity.

Keywords: Measurement Problem; Born Rule; Everettian Interpretation; Self-Locating Uncertainty; Epistemic Separability Principle; Quantum Foundations

INTRODUCTION

Quantum Mechanics has been around for nearly a century, and as we mark its 100th anniversary, it remains one of the most successful theories in physics. Despite its prominence, a core issue in the foundations of quantum mechanics remains unsolved: the “measurement problem”. The measurement problem concerns why

only a single outcome is observed after quantum measurements, despite the quantum mechanics allowing for multiple possibilities. Over the years, many have attempted to tackle this issue through mathematical and philosophical approaches. Among these efforts, one particular derivation by Charles T. Sebens and Sean Carroll has received significant attention. They offer a derivation of the Born Rule, a fundamental element of Quantum Mechanics, which in turn provides a potential solution to the measurement problem. This paper delves into the Sebens-Carroll approach and explores both the need to derive the Born Rule and the broader implications their work has on modern developments in Quantum Mechanics.

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Schrödinger Equation

Quantum mechanics describes how particles and systems behave at the smallest scales. At its core lies the Schrödinger equation, a foundational principle that governs how a system's wavefunction (its complete quantum state as a function of momentum, position, and spin) changes over time. This change over time, also called time-evolution, is unitary, meaning it's smooth, continuous, and conserves probability. Smooth and continuous time-evolution implies that quantum states do not undergo sudden jumps or discontinuities as they change over time. Probability conservation, meanwhile, implies that the total probability of all possible outcomes always remains the same, even as the system evolves.

Under the quantum mechanics framework, a system can exist in a superposition, where multiple possible states coexist simultaneously. According to the Schrödinger equation, as a superposition evolves to some future state (even if it undergoes processes like a measurement), its evolution should remain unitary. This is what should happen in theory: smooth, continuous, and probability-conserving evolution that follows the criteria set by the Schrödinger equation.

Copenhagen Interpretation

In practice, quantum systems do not always appear to evolve unitarily according to the Schrödinger equation. According to the Copenhagen interpretation, the standard textbook interpretation of quantum mechanics, when we measure a quantum system in a superposition, the superposition “collapses” (1), in that the superposition of states instantly transitions to a single state. This is what is referred to as “wavefunction collapse”. For example, if you catch a rapidly spinning coin, analogous to measuring a quantum system in a superposition, the coin is observed to be either heads or tails, never both. By measuring the system, it transitions from a superposition to a single state instantaneously. This immediate transition is not smooth or continuous (indicating non-unitarity), and thus is not described by the Schrödinger equation. In addition to its non-unitarity, the Copenhagen interpretation does not provide a sufficient account for where probabilities come from—that is, what governs the likelihood of observing one outcome over another? This discrepancy between what happens in reality (by the Copenhagen interpretation) and what should happen in theory (by the Schrödinger equation) is known as the measurement problem (2).

Everettian Quantum Mechanics (EQM)

A compelling solution to the Measurement Problem

comes from EQM, an interpretation of quantum mechanics proposed by Hugh Everett in 1957. EQM introduces the Many-Worlds Interpretation (MWI), which rejects wavefunction collapse entirely (3). Instead, it asserts that following a measurement, all possible outcomes actually occur, but each in separate and non-interacting branches of the universe. For example, if you catch a spinning coin, EQM says the universe splits. One branch of the universe registers heads, and other registers tails. Both outcomes are real, but whichever branch of the universe you find yourself on determines the outcome you observe (4). This universe-splitting, multiple worlds mechanism is where the MWI gets its name.

Because EQM involves no collapse, evolution of the wavefunction in the context of EQM is smooth and continuous, aligning with the premises of the Schrödinger Equation. This immediately solves half of the measurement problem by eliminating collapse entirely.

However, one major aspect remains: we need a sufficient account for probabilities – i.e. if all outcomes occur, where do probabilities come from? Why do we perceive some outcomes more often than others?

Born Rule

Enter the Born Rule. The Born Rule is a foundational principle in quantum mechanics which relates the wavefunction (symbolized Ψ) of a system with the probabilities of different outcomes (5). Specifically, it relates the probability of outcomes with the square of components within the wavefunction. The Born Rule is not only applicable in theory, it has extensive experimental support, as real quantum systems consistently produce outcomes in line with its statistical predictions.

Why Derive the Born Rule?

The Born Rule has seen a lot of success, due to the fact that it consistently and accurately predicts the outcomes of quantum experiments. Hence, the Born Rule is typically assumed rather than derived, therefore it lacks a deeper understanding for why it works. Noted physicist, James Hartle, argues that deriving the Born Rule is not just a mere academic exercise, as it helps clarify how probability naturally arises from the theory, and strengthens the internal consistency of quantum mechanics (10).

A derivation within EQM is especially convenient since the Born Rule provides a solution to half of the measurement problem (by providing a sufficient account of probabilities), and EQM provides a solution to the second half (by ignoring wavefunction collapse). Therefore, a derivation of the Born Rule within the context of EQM

would appear to fully resolve the measurement problem.

Many have attempted to derive the Born rule within the context of EQM, but the approach used by Charles T. Sebens and Sean Carroll has received the most attention.

METHODOLOGY OF THE SEBENS-CARROLL PROOF

This proof utilizes unique methods and tools to derive the Born Rule. We will briefly consider these methods and tools prior to reviewing their use in the Born rule derivation—these include entanglement, decoherence, Self-Locating Uncertainty (SLU), the Epistemic Separability Principle (ESP), and unitary operators.

Entanglement

Entanglement is a phenomenon in quantum mechanics where two or more particles, separated by any amount of distance large or small, have an effect on each other. Their properties remain interdependent, even when separated across vast distances. Once entangled, a change to one particle, such as a measurement, will have an instantaneous effect on its counterpart. This phenomenon was first coined by Albert Einstein in the Einstein-Podolsky-Rosen Paradox in 1935 as “spooky action at a distance” (6). The implication of “spooky action” was that the mechanism underlying this mysterious interaction was not known at that time to the field of physics.

Decoherence

Decoherence occurs when a quantum system becomes entangled with its environment (7). This process makes the outcomes of a measurement appear classical (like what we observe in real life), and mutually exclusive (no two outcomes can happen at the same time). In the MWI, decoherence is what gives rise to distinct branches and outcomes. It thus explains why the universe appears to “split” without undergoing collapse.

SLU

SLU was originally postulated as a philosophic principle. From a philosophical standpoint, SLU happens when you know all the facts about the world but still don't know your own place or identity within it (15).

Sebens and Carroll extend this principle to fit within the context of EQM. When an observer performs a measurement, he then becomes entangled with the system. The universe then branches, and copies of the observer appear on each branch. Before they can register the result and perceive which branch they are on, due to limitations

in human perception, the observer is unsure of where they are in the multiverse (8). This uncertainty about which branch the observer occupies is known as SLU.

ESP

In the MWI, the ESP says that any observer's actions and decisions are only influenced by their own branch. Carroll and Sebens utilize ESP to explain that probabilities in any one branch are unrelated and un-influenced by events in other branches when the observer is in a state of SLU. (8). This principle, combined with SLU, is key to their derivation of the Born Rule.

Unitary Operators

One additional mathematical operation that is crucial to this derivation is the “unitary operator”. Unitary operators can transform quantum states in a way that preserves total probability. Unitary operators allow us to re-express or manipulate quantum systems for the purposes of a derivation, without violating any rules of quantum mechanics. Unitary operators are critical to the Born rule derivation because it allows us to transform the environment in a manner so that we can more easily work with it. This transformation has the important quality of not being either creative or destructive. It preserves all the important qualities of a state, like its probability.

DERIVATION FOR EQUAL AMPLITUDES

Carroll and Sebens begin with a derivation for an equal amplitude case. Amplitudes are coefficients that appear in the wavefunction equation of a system. Closely tied to the Born Rule, amplitudes are directly related to probabilities. Specifically, the square of these amplitudes denotes the probabilities of their respective states. An equal amplitude case means that the amplitudes associated with states are equal, indicating equal probabilities of each outcome. Such may be the case with a fair coin flip - 50% heads, and 50% tails.

Beginning their derivation, Sebens and Carroll set up a situation including the following: a particle that could be either spin up (\uparrow) or down (\downarrow), an observer (O), and the environment (Ω) consisting of everything else—such as detectors, the air, physical surroundings, etc. (Figure 1). Next, they apply a unitary operator to mathematically reconfigure the environment without changing the core qualities or probabilities of the system. This can be understood like rearranging furniture in a room without adding or removing anything from it—the setup might be different, but the room is the same. This rearrangement

doesn't change any probabilities of outcomes. Continuing the derivation, Sebens and Carroll bring in a standard two-sided coin, with heads (H) and tails (T) (Figure 2).

Next, they entangle the coin with the spin up/down particle, so that their states are linked with each other. (Figure 3). In one version (Ψ_1), spin up is linked with heads,

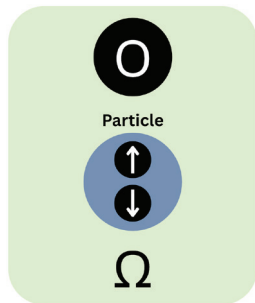


Figure 1. An observer O, and a particle with spin up and down states (↑, ↓) exists within an environment Ω (green box). They are all entangled, meaning that any change in one affects the others.

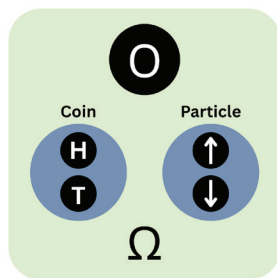


Figure 2. A coin with heads (H) and tails (T) states is added to the setup in Figure 1.

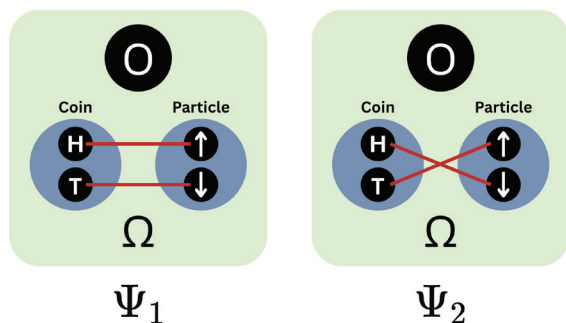


Figure 3. Two scenarios, Ψ_1 and Ψ_2 , each depicting entanglement between a coin and a particle. Ψ_1 : H is entangled with ↑ and T is entangled with ↓. Ψ_2 : H is entangled with ↓ and T is entangled with ↑.

and spin down is linked with tails. In another version (Ψ_2), the pairing is swapped. Although the entanglement pairings are different between Ψ_1 and Ψ_2 , the probabilities remain the same. Because ↑ in Ψ_1 and ↓ in Ψ_2 are both entangled with heads, we can conclude that the probability of ↑ in Ψ_1 and the probability of ↓ in Ψ_2 are the same. Furthermore, because the probability of H in a fair coin is 50%, the spin entangled with heads also carries a 50% probability. Finally, because the sum of all probabilities is 100%, we can conclude the opposite spin also has 50% probability. This result matches exactly with the Born Rule prediction for an equal amplitude case.

DERIVATION FOR UNEQUAL AMPLITUDES (PART 1)

However, we can't always expect outcomes to be equally likely. Coins can be weighted and dice rigged to yield uneven probabilities. Similarly, in quantum systems probabilities may have an unequal distribution. Therefore, Sebens and Carroll consider the Born Rule derivation for the case of unequal amplitudes.

Assume a particle with two states, spin up and spin down. However, in this case, spin up and spin down do not have equal probabilities. Let us assume that the spin down branch has an amplitude greater by a factor of $\sqrt{2}$. If we follow the Born Rule, we expect to see spin down to be twice as likely as spin up, since we square the amplitudes. But how do Sebens and Carroll show this?

Applying a unitary operator, they use a clever method that turns the case of unequal amplitudes into one of equal amplitudes. Specifically, they split the branch with greater weight – in this case, the spin down branch. Now, the wavefunction has three equal branches, one for spin up and two for spin down. Since the branches now all have equal amplitudes, they apply a technique called branch counting, where one literally counts how many branches go with each outcome. Since there are two spin down branches and one spin up branch, the probability of spin down becomes $\frac{2}{3}$, and the probability of spin up becomes $\frac{1}{3}$.

According to the Born Rule, the probabilities of outcomes are given by their amplitudes squared. In the setup of the above experiment, the spin down amplitude was greater by a factor of $\sqrt{2}$. Therefore, applying the Born Rule, the probability would be given by the square of $\sqrt{2}$, or simply 2. In other words, the Born Rule predicts $\frac{2}{3}$ probability of spin down, and $\frac{1}{3}$ probability of spin up, which matches precisely with the result achieved by branch counting.

However, at this point, the derivation can still be criticized as being circular in logic. We started off with amplitudes we already knew, and simply made the math align. This is where the second part of the derivation comes in.

DERIVATION FOR UNEQUAL AMPLITUDES (PART 2)

In order to prove we are not simply using the Born Rule to prove the Born Rule, Sebens and Carroll take an additional step. Again, they split the environment into subsystems. This time, they use more complex systems: a set of three states with numbers $\{+,0,-\}$, and another set of three states with card suits $\{\heartsuit,\spadesuit,\clubsuit\}$ (Figure 4).

They once again create two setups, where spin states of the particle are entangled with the subsystems.

In the first setup Ψ_1 (Figure 5), spin up is linked to $(+, \heartsuit)$ (as depicted by green lines). Spin down is split into two branches: one branch linked to $(-, \spadesuit)$ (as depicted by blue lines), and another branch linked to $(0, \clubsuit)$ (as depicted by red lines).

In the second setup Ψ_2 (Figure 6), the entanglements are swapped. Spin up is now linked to $(-, \spadesuit)$ (green lines). Spin down has one branch linked to $(0, \heartsuit)$ (blue lines), and another branch linked to $(+, \clubsuit)$ (red lines).

These two setups, despite appearing different, are mathematically identical. By the same logic used in the equal amplitude derivation, the probabilities across both setups can be compared. The probability of spin up, $-$, and \heartsuit is the same in both scenarios. Both branches lead to the same observations. By tracing how each component (spin $\{\uparrow,\downarrow\}$, number $\{+,0,-\}$, and card $\{\heartsuit,\spadesuit,\clubsuit\}$) is linked in both setups, Sebens and Carroll confirm that each individual

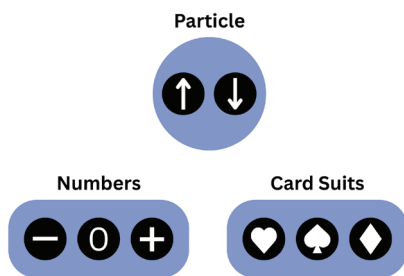


Figure 4. This setup includes three systems. The first is a particle, with spin up (↑) and spin down (↓) states. The second is a system containing $\{+,0,-\}$, referred to as “numbers” for convenience. The third is a system containing card suits $\{\heartsuit,\spadesuit,\clubsuit\}$.

branch has equal weight.

Now that all branches are equal, simple branch counting can be applied. Since there are two branches connected to spin down, and one connected to spin up, we can deduce that the spin down outcome has a $\frac{2}{3}$ probability, while the spin up branch has a $\frac{1}{3}$ probability. This again aligns perfectly with the probabilities predicted by the Born Rule. Therefore, this example shows how the Born Rule can be derived, for the case of unequal amplitudes.

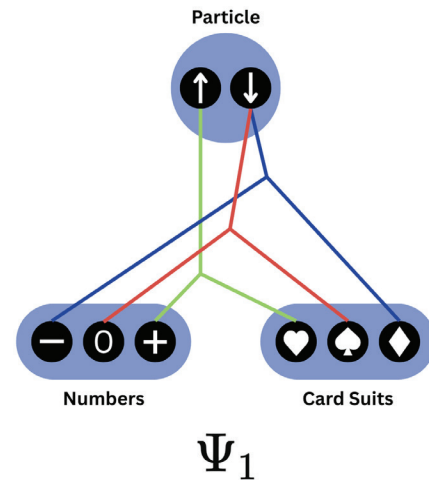


Figure 5. Ψ_1 shows a setup in which spin up or spin down is entangled with numbers and card suits, as depicted by the green, red, and blue lines. Spin up is linked to $+$ and \heartsuit (green lines). Spin down is linked to $-$ and \spadesuit (blue lines), as well as 0 and \clubsuit (red lines).

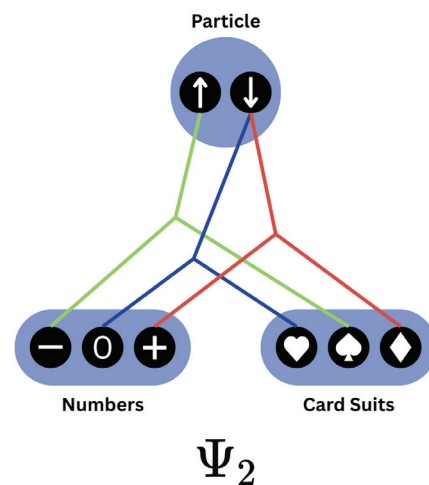


Figure 6. Ψ_2 shows a setup in which spin up or spin down is entangled with numbers and card suits, as depicted by the green, red, and blue lines. Spin up is linked to $-$ and \spadesuit (green lines). Spin down has one branch linked to 0 and \heartsuit (blue lines), and another linked to $+$ and \clubsuit (red lines).

CONCLUSION

This paper describes Sebens and Carroll's method for deriving the Born Rule under the framework of EQM. They use unitary operators and ESP, as well as a unique argument utilizing different entanglement combinations to transform the state to a point where simple branch counting can be applied. In doing so, they provide logical, visually apparent, and mathematically sound proof for the Born Rule. Furthermore, this proof is valid for both cases of equal and unequal amplitudes— i.e. equal and unequal probability of outcomes.

Though Sebens and Carroll's method of deriving the Born rule using SLU and ESP has been successful, it has also faced criticism. For example, physicist Adrian Kent (11) argued that SLU does not necessarily imply the existence of multiple observer copies across different branches of the universe. A single observer with no SLU remains an alternative. He also challenges the notion of branching, arguing that if branches aren't precisely defined, then SLU cannot be applied effectively, weakening the basis of Sebens and Carroll's approach. Richard Dawid and Simon Friederich criticize the ESP, arguing that the principle isn't truly based on a general principle, but partly depends on assuming the Born rule's validity (12).

Emily Adlam argued that SLU doesn't give a clear or consistent way to assign probabilities. Depending on how one defines or counts different versions of the observer, impact on the probability assignments can vary (13). As there's no objective rule for making these determinations, the approach lacks clarity and validity. These critiques demonstrate that although the Sebens and Carroll approach is robust, alternative views exist.

In addition to the previously mentioned criticisms, refinements to the Sebens Carroll method have been proposed. Sebens and Carroll's derivation uses simple numbers. However, in the event that there are irrational numbers in the branch amplitudes, application of their method becomes complex. To address this, one proposed solution is to approximate the irrational amplitudes using simpler, rational values.

Another limitation stems from their reliance on branch counting. In the final steps of their derivations, Sebens and Carroll utilize simple branch counting to assign probabilities. They do so by counting the number of branches connected to either spin up or spin down. However, defining what a branch is can be arbitrary and subjective, and branches can always be further split, meaning there is an indeterminate number of branches.

Thus, it is simpler to count in terms of ratios of branches, rather than absolute numbers of branches (16).

Moreover, the Sebens Carroll approach relies heavily on ESP and SLU, both of which are rooted in philosophical principles, which raises questions regarding the strength of the derivation from a physics standpoint. In addition, SLU and ESP also have no experimental backing. These aspects reveal limitations to the Sebens-Carroll approach and identify opportunities for further research. Finally, understanding the measurement problem has been used in other areas of theoretical and experimental physics – for example quantum gravity (14).

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DECLARATION OF CONFLICT OF INTERESTS

The authors declare that there are no conflicts of interest regarding the publication of this article.

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